International Research Journal of Multidisciplinary Scope (IRJMS), 2024; 5(1): 379-386

Original Article | ISSN (O): 2582-631X DOI: 10.47857/irjms.2024.v05i01.0226

Applications of Labeling in Hypergraph

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Abstract

One of the major and rapidly growing fields of graph theory is hypergraph. A hypergraph is the generalization of the graph. In a simple graph, each edge is identified by 2 end vertices. Also, in a hypergraph, each edge is identified by 1, 2, or more than 2 vertices. Labeling hypergraph plays a major role in many fields. Many researchers define various labeling techniques for hypergraphs. One such labeling is the prime labeling of a hypergraph which defines that each vertex is labeled with a number from 1 to |V| so that the gcd of labels within each hyperedge is 1. The present study examines a new labeling technique namely, relatively prime labeling and relatively prime edge labeling of a hypergraph. Relatively prime labeling states that each vertex is assigned with a distinct number from 1 to |V| so that the labels of vertices of each edge are pairwise relatively prime. Likewise, relatively prime edge labeling of a hypergraph is defined in such a way that each edge is assigned with the labels from 1 to |E|, satisfying the condition that the labels of edges containing each vertex are relatively prime. Also, characterization of relatively prime labeled and relatively prime edge labeled hypergraphs is found in this study. And, for what values of m, n, and k, m-node k-uniform hyperpath and m-node k-uniform hypercycle admits relatively prime labeling. Finally, various applications of hypergraph and hypergraph labeling are illustrated with an example. Defining a new labeling technique for hypergraphs with innovative application makes the present study novel for every researcher.

Keywords: Relatively prime labeled hypergraph, Relatively prime edge labeled hypergraph, Hyperpath, Hypercycle, Incidence hypergraph.

Introduction

Euler was the first to deploy a graph model to solve the Seven Bridges of Königsberg question in 1736. Sylvester first used the word "graph" in 1878. Graphs are widely used in many fields. Although graphs have undergone many developments in the first half of the 20th century, there is still much to learn about them. The hypergraph concept was introduced by Berge in the 1960s which is the generalization of graphs (1). Some situations cannot be represented by a typical graph network, so hypergraph plays an important role in handling those situations. Graphs accommodate only pairwise connections, while hypergraphs maintain multiple connections, making them a perfect choice for representing collaborative networks and other scenarios.

One important development in the study of hypergraphs was the introduction of hypergraph coloring. In traditional graph theory, graph coloring is the assignment of colors to vertices such that no two adjacent vertices share the same color. In hypergraphs, the concept of coloring was extended to edges, leading to the notion of hypergraph coloring. Several coloring concepts, such as vertex coloring, and hypergraph incidence coloring (2), were adapted to hypergraphs. Another devolopment in hypergraph is domination in hypergraph (3, 4). Hypergraphs find applications in various areas, such as machine learning, social network analysis, and data mining (5). They provide a more adaptable way of representing complex relationships between entities, leading to more accurate and efficient analysis.

Among the enormous variety of concepts in graph theory, the idea of labeling graphs has become very popular. For a variety of high-technology applications, graph labeling offers practical mathematical models. Any mapping that converts a certain set of graph elements to a particular set of numbers under specific restrictions is known as labeling or valuing a graph. There are various labeling techniques defined by various researchers for hypergraph, such as (α, β)-labelling method for k-uniform hypergraph (6), Antimagic vertex labeling of hypergraphs (7),

(Received 08th November 2023; Accepted 13th January 2024; Published 30th January 2024)

Super Edge-Magic Labeling of 5-Uniform and 6-Uniform Hypergraphs (8, 9), on exclusive sum labeling of hypergraphs (10), On Edge Product Hypergraphs (11), Sum labeling of cycle hypergraphs (12), On special numberings of hypergraphs (13) and so on.

In prime labeling for a simple graph (14), the vertices can be assigned with different numbers ranging from 1 to |V| such that the labels of vertices along each edge are relatively prime. The following is a natural extension of hypergraphs, where edges may have more than two vertices.

In prime labeling for a hypergraph, the vertices can be given with different numbers ranging from 1 to |V| so that the gcd of the labels within each hyperedge is 1. These facts became the basis for the subsequent extensions of the prime labeling definitions. This paper discusses the relatively prime labeling and relatively prime edge labeling of hypergraphs.

Preliminaries

We referred to Berge (1) for all terminology and notation in the hypergraph. This paper only discusses nonempty, isolate-free, and simple hypergraphs. A hypergraph is a set, $\mathcal{H} = (\mathrm{V}, \mathcal{E})$. Here, V stands for the vertex set and $\mathcal{E} \subseteq \mathrm{P(V)} - \{\emptyset\}$ for the hyperedge.

The number of hyperedges connected to a vertex is the degree of a vertex in a hypergraph. The number of vertices in the hyperedge is the hyperedge degree. A hypergraph is referred to as k - uniform if each hyperedge has precisely K vertices. A hypergraph is said to be D-regular if each vertex has a degree D. The incidence graph of a hypergraph is represented as a bipartite graph, where a vertex in the hypergraph belongs to a partite and the edges in the hypergraph belong to another partite. The edge joining the two partite depicts that, a vertex in one partite belongs to the edge in the other partite.

A k_1 −regular k_2 −uniform hypergraph (15, 16) is a hypergraph with n vertices and m hyperedges, such that each hyperedge contains k_2 vertices and each vertex is in k_1 hyperedges. If $k_2 = 2$, then the hypergraph is simple. A graph obtained from a cycle C_n by adding a pendent edge to each vertex on the cycle is said to be the sunlet graph S_n.

A hypergraph has a prime labeling if its vertices are labeled with 1 to |V| such that the gcd of labels of vertices within each hyperedge is 1. Given a hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$, where $V = \{v_1, v_2, \dots, v_n\}$ and $E = \{e_1, e_2, \dots, e_m\}$, the incidence matrix $M = [m_{ij}]$ of H is an $n \times m$ binary matrix such that $m_{ij} = 1$ if and only if $v_i \in e_j$(1, 2).

Section 3 of this paper defines the concept of relatively prime labeling for hypergraphs, and provides a detailed discussion on the existence and non-existence of such labeled hypergraphs. The concept is then extended to edges, relatively prime edge labeling is defined, and special classes of graphs having relatively prime edge labeling are examined.

Section 4 begins by explaining the general applications of hypergraphs, before focusing on the specific application of relatively prime labeling for hypergraphs. An example is provided to illustrate this application.

Proposed labeling technique

Hypergraph labeling is a technique used to assign labels or numbers to the vertices and edges of a hypergraph. A hypergraph is a generalization of a graph in which each edge can connect more than two vertices. In hypergraph labeling, the labels assigned to vertices and edges follow certain rules or constraints that depend on the type of labeling being used.

One such example of hypergraph labeling is prime labeling (1), where each vertex is labeled with a number from 1 to the total number of vertices so that the greatest common divisor of labels within each hyperedge is 1. This means that the labels assigned to vertices within each edge have no common factors other than 1.

Relatively prime labeling of hypergraphs

This section introduces relatively prime labeling of hypergraphs in detail. Prime labeling of hypergraph labels the vertices in such a way that the gcd of labels of vertices in every hyperedge is 1. By extending the above definition, relatively prime labeling of a hypergraph is defined in a manner that, every pair of vertices in every hyperedge receives relatively prime labels.

Definition

A hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ has a relatively prime labeling if its vertices can be assigned with distinct numbers from 1 to |V|, such that the labels of the vertices on each edge are relatively prime.

In other words, $\mathrm{L} : \mathrm{V}(\mathcal{H}) \rightarrow \{1, 2, \cdots\cdots, |\mathrm{V}|\}$, such that $\gcd(L(v_i), L(v_j)) = 1$, for each $e_i \in \mathcal{E}$ and for every pair $v_i, v_j \in e_i$ $with\ i \neq j$.

Example

For a hypergraph, $\mathcal{H} = (\mathrm{V}, \mathcal{E})$,where V denotes the vertices in the hypergraph and $\mathcal{E}$ denotes the edges in the hypergraph such that,

$\mathrm{V}(\mathcal{H}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$\mathcal{E} = \{e_1, e_2, e_3, e_4, e_5\}$ where,

$e_1 = \{v_1, v_2, v_3\}$, $e_2 = \{v_2, v_3, v_4\}$, $e_3 = \{v_5, v_6, v_7\}$,

$e_4 = \{v_3, v_4, v_5\}$, $e_5 = \{v_1, v_5, v_7\}$

Define, $\mathrm{L} : \mathrm{V}(\mathcal{H}) \to \{1, 2, \cdots\cdots, 7\}$, by

$\mathrm{L}(v_1) = 2, \mathrm{L}(v_2) = 3, \mathrm{L}(v_3) = 1, \mathrm{L}(v_4) = 4,$

$\mathrm{L}(v_5) = 5, \mathrm{L}(v_6) = 6, \mathrm{L}(v_7) = 7.$

In the above labeling, every pair of labels of vertices in each edge is relatively prime. Thus, $\mathcal{H}$ is the relatively prime labeled hypergraph.

Proposition

Every relatively prime-labeled hypergraph is the prime-labeled hypergraph, but the converse is not true.

Proof

As in relatively prime labeled hypergraph, the labels of vertices corresponding to each edge are pairwise relatively prime. Hence, the gcd of labels of vertices corresponding to each edge is 1, making it a prime labeled hypergraph. But since the gcd of labels of vertices corresponding to each edge is one, which is not sufficient to be pairwise relatively prime, the converse fails.

The following theorem finds the existence and non-existence of relatively prime-labeled hypergraphs, under certain conditions.

Theorem For $n \leq 14$, there exists a relatively prime labeled hypergraph having a hyperedge with degree $\left\lfloor \frac{n}{2} \right\rfloor$.

Proof.

Suppose, there exists a relatively prime labeled hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ with $n > 14$ vertices and having an edge with, $|e| = \left\lfloor \frac{n}{2} \right\rfloor$. In a relatively prime -labeled hypergraph, the vertices in each edge are labeled with relatively prime pairs. Thus, the maximum possible prime pairs less than n will be $\{1, 2, 3, 5, 7, 11, 13, 17, 19, 23 \ldots\ldots\}$, which will be less than $\left\lfloor \frac{n}{2} \right\rfloor$ for $n > 14$.

Hence, for $n > 14$ labeling the $\frac{n}{2}$ vertices in an edge with relatively prime pairs is not possible. Which contradicts the fact that, $\mathcal{H}$ is a relatively prime labeled hypergraph.

Theorem Let $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ be a hypergraph having a hyperedge with degree $n - 1$, then $\mathcal{H}$ is not a relatively prime labeled hypergraph.

Proof.

In a relatively prime labeled hypergraph, the labels of vertices on each edge are pairwise relatively prime. Since there is an edge with n-1 vertices, it is not possible to label vertices with pairwise relatively prime.

Theorem Let a hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ be a k – uniform connected hypergraph, then there exists a relatively prime labeled graph only if $k \leq 3$.

Proof.

As $\mathcal{H}$ is a k – uniform connected hypergraph, every edge of $\mathcal{H}$ has k vertices and is adjacent to some other edges. Suppose $k > 3$ (say m), then every edge has m vertices. By labeling the vertices of an edge e with {1,2,3,5,7...m} which will be pairwise relatively prime, then the vertices in the remaining edges receive the labels {4,6, 8……m} which will not be pairwise relatively prime. Hence the proof.

For m, n, and $k \geq 2m$, the m –node k – uniform hyperpath(15) is a hypergraph with m vertices and n hyperedges with each hyperedge having k vertices and

$$e_i = \begin{cases} \bigcup_{j=1}^{m} \{x_{i,j}, x_{i+1,j}\} & if\ k = 2m \\ \bigcup_{j=1}^{m} \{x_{i,j}, x_{i+1,j}\} \cup \{y_{i,1}, y_{i,2} \ldots y_{i,k-2m}\} & if\ k > 2m \end{cases}$$

for $i \in [1, n]$

The next theorem finds for what values of m, n, and k , $m -$ node $k -$ uniform hyperpath is relatively prime labeled.

Theorem An m – node k – uniform hyperpath $mp_n^{(k)}$ is relatively prime labeled for $m = 1$ and $k = 3$.

Proof.

The proof of the theorem is obvious, by labeling the vertices of the hyperpath consecutively from 1 to n.

From the above definition, m – node k – uniform hypercycle is defined, for any $m \geq 1$, $n \geq 3$, and $k \geq 2m$ having m vertices and n hyperedges with each hyperedge having k vertices and

$$e_i = \begin{cases} \bigcup_{j=1}^{m} \{x_{i,j}, x_{i+1,j}\} & if\ k = 2m \\ \bigcup_{j=1}^{m} \{x_{i,j}, x_{i+1,j}\} \cup \{y_{i,1}, y_{i,2} \dots \dots y_{i,k-2m}\} & if\ k > 2m \end{cases}$$

for $i \in [1, n]$ and $x_{n+1,j} = x_{1,j}$ for $j \in [1, m]$

Corollary An m $-$ node $k -$ uniform hypercycle $mc_n^{(k)}$ is relatively prime labeled for $m = 1\ and\ k = 3$.

Theorem For any $n > 3$, with $n - 1$ edges there exists a relatively prime labeled 3 – uniform hypergraph.

Proof.

To construct a relatively prime labeled 3 – uniform hypergraph $\mathcal{H}$ with n vertices and n – 1 edges, label the vertices of $\mathcal{H}$ such that each vertex v_i is labeled with i.

We now prove this theorem by induction on the number of vertices, n. For $n = 4$, the theorem holds true as given in figure 1.

Assume that, the theorem holds true for $n \leq k$ vertices, that is there exists a relatively prime labeled 3 – uniform hypergraph with k vertices. We shall now prove for $n = k + 1$ vertices. Let $\mathcal{H}$ be a hypergraph with $k + 1$ vertices and k edges, it is enough to prove that $\mathcal{H}$ is a relatively prime labeled 3 – uniform hypergraph. By assumption, there exists a relatively prime-labeled 3 – uniform hypergraph with k vertices, now by adding a vertex v_{k+1} having label $k + 1$ and an edge e_k containing the vertices v_k , v_{k+1} and v_1 results a relatively prime labeled 3 – uniform hypergraph, as the edge e_k contains relatively prime labels $\{ k, k + 1, 1\}$.

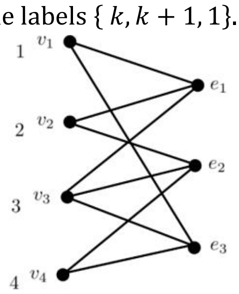

Figure – 1

Theorem

Let the hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ with $2m + 1$ vertices and m hyperedges having $| e_1 \cap e_2 \cap e_3 \cdots \cap e_m| = 1$and the maximum edge degree of a hyperedge is $\pi(2m + 1) + 1$, then $\mathcal{H}$ is relatively prime labeled hypergraph.

Proof.

Let v_1 be the common vertex that exists in all the edges $e_i, i = 1, 2, \dots, m$. Label the vertex v_1 of the hypergraph $\mathcal{H}$ as 1.

Case 1: $|e_i| = 3\ , for\ all\ i = 1, 2, \dots, m$

For $|e_i| = 3\ , i = 1, 2, \dots, m$, then $\mathcal{H}$ is a 3- uniform hypergraph with vertices, $\{v_1,\ v_2, \cdots, v_{2m+1}\}$. Edges in the hypergraph $\mathcal{H}$ are as follows:

$e_1 = \{v_1, v_2, v_3\}, e_2 = \{v_1, v_4, v_5\},$ $e_3 = \{v_1, v_6, v_7\}, \cdots \cdots, e_m = \{v_1, v_{2m}, v_{2m+1}\}$

Now, label the vertices of each v_i as i having each edge a pairwise relatively prime label.

Case 2: $|e_i| \neq 3\ ,\ for\ some\ i = 1, 2, \dots, m$

Let the vertices in the hypergraph be, $\mathrm{V} = \{v_1, v_2, \cdots, v_{2m+1}\}$ and label the vertices of each v_i as i. For $|e_i| \neq 3$, there exists atleast one hyperedge having $|e_i| > 3$. As the maximum edge degree is $\pi(2m + 1) + 1$, and hence if there exists an hyperedge with $\pi(2m + 1) + 1$ vertices, which can be labeled with $\{ p_1, p_2, \cdots, p_{\pi(2m+1)}, 1\}$. Hence, the hyperedge with maximum degree is pairwise relatively prime. In this case, the remaining hyperedge contains atleast 2 or 3 vertices, which will be obviously relatively prime.

Corollary Let $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ be a d- uniform hyperstar with n vertices, then $\mathcal{H}$ is relatively prime labeled hypergraph for all values of n and for $d = 3$.

Proof.

The result follows directly from the above theorem.

Relatively Prime Edge Labeling of Hypergraphs

As a continuation of the previous topic, by considering edges into the account this section introduces a new labeling namely, relatively prime edge labeling of a hypergraphs. In relatively prime labeling of hypergraph, the vertices are labeled from 1 to |V| such that every pair of adjacent vertices receives relatively prime labels. Similarly, in relatively prime edge labeling the edges are labeled from 1 to |E| with the condition that edges containing the each vertex should be pairwise relatively prime.

Definition

A hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ is said to be a relatively prime edge labeling if its edges are labeled from 1 to |E|, such that the labels of the edges containing the vertex v are pairwise relatively prime.

Example

For a hypergraph, $\mathcal{H} = (\mathrm{V}, \mathcal{E})$,where V and $\mathcal{E}$ represents the vertices and edges of hypergraph such that,

$\mathrm{V}(\mathcal{H}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$\mathcal{E} = \{e_1, e_2, e_3, e_4, e_5\}$ where, $e_1 = \{v_1, v_2, v_3\}$, $e_2 = \{v_2, v_3, v_4, v_7\}$, $e_3 = \{v_5, v_6, v_2\}$, $e_4 = \{v_3, v_4, v_5, v_6\}$, $e_5 = \{v_1, v_4, v_7\}$

Define, $\mathrm{L} : \mathrm{E}(\mathcal{H}) \rightarrow \{1, 2, \dots, 5\}$, by $\mathrm{L}(e_1) = 1, \mathrm{L}(e_2) = 3, \mathrm{L}(e_3) = 2, \mathrm{L}(e_4) = 5, \mathrm{L}(e_5) = 4$.

Hence by the defined labeling, edges containing the vertex $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ are ($e_1, e_5), (e_1, e_2), (e_1, e_2, e_4), (e_2, e_4, e_5), (e_3, e_4), (e_3, e_4)$ $and\ (e_2, e_5)$, with labels {(1,4), (1, 3), (1,3,5), (3,4,5), (2,5), (2,5) and (3,4)} respectively, which are pairwise relatively prime. Thus, $\mathcal{H}$ is the relatively prime edge labeled hypergraph.

Construction of 5- uniform and 6- uniform hypergraph from the simple graph is as follows (5): Let G be a simple graph with the vertex set $\{v_1, v_2, \dots, v_p\}$ and the edge set $\{e_1, e_2, \dots\dots e_q\}$. Construct an additional vertex set $\{v'_1, v'_2, \dots, v'_p\}$. For each $e_k = v_i v_j$ of G, define $E_k = \{e_k, v_i, v'_i, v_j, v'_j\}$. Then, a hypergraph whose vertex set and hyperedge set are $\bigcup_{k=1}^{q} E_k$ and $\bigcup_{k=1}^{q} \{E_k\}$, respectively, is called the 5-uniform hypergraph generated by G and denoted by $H^{(5)}(G)$.

Let G be a simple graph having vertices, edges as $\{v_1, v_2, \dots, v_p\}$ and $\{e_1, e_2, \dots\dots e_q\}$.

Construct an additional vertex set $\{v'_1, v'_2, \dots, v'_p\}\ and\ \{e'_1, e'_2, \dots\dots e'_q\}$. For each $e_k = v_i v_j$ of G, define $E_k = \{e_k, e'_k, v_i, v'_i, v_j, v'_j\}$. Then, a hypergraph whose vertex set and hyperedge set are $\bigcup_{k=1}^{q} E_k$ and $\bigcup_{k=1}^{q} \{E_k\}$, respectively, is called the 6-uniform hypergraph generated by G and denoted by $H^{(6)}(G)$.

The following theorem finds the existence of a relatively prime edge - labeled hypergraph for some classes of graphs.

Theorem: A 5 - uniform hypergraph $H^{(5)}(G)$ generated from the sunlet graph $G = (p, q)$ is relatively prime edge labeled hypergraph.

Proof.

5 - uniform hypergraph is obtained from the sunlet graph by the above-mentioned constructions, that contain q hyperedges, say $\{E_1, E_2, \dots, E_q\}$. Here, $\{E_2, E_4, \dots,\}$ represents the hyperedge which corresponds to the pendant vertex. Now, label the hyperedge in such a way that, $L(E_i) = i, i = 1,2, \dots, q$. Also, for each, i there exists two vertices v_i, v'_i , which is in 3 hyperedges and the labeling corresponding to those vertices will be $\{2m - 1, 2m, 2m + 1\}$ for some m, which is also pairwise relatively prime. Hence it is clear that, 5 - uniform hypergraph obtained from the sunlet graph is a relatively prime edge labeled hypergraph.

Corollary: A 6 - uniform hypergraph $H^{(6)}(G)$ generated from the sunlet graph $G = (p, q)$ is relatively prime edge labeled hypergraph.

Theorem: An m -node k- uniform hyperpath $mp_n^{(k)}$ and m -node k- uniform hypercycle $mc_n^{(k)}$ are relatively prime edge labeled hypergraph.

Proof.

The proof of the following theorem is straight forward, by labeling the edges of hyperpath and hypercycle consecutively from 1 to n, where each vertex is in 2 or 1 hyperedge. If a vertex is in 2 hyperedges it will be a consecutive hyperedge with labels containing consecutive numbers, and hence relatively prime.

In the next theorem, for what values of d, d – uniform strong cycle (7) is relatively prime edge labeled is found.

Theorem: Every d – uniform strong cycle $c_n^d = (\mathrm{V}, \mathcal{E})$ is relatively prime edge labeled graph only for $d = 2$.

Proof. The proof is straight forward.

Theorem: Let $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ be a d- uniform hyperstar having n vertices, then $\mathcal{H}$ is relatively prime edge labeled hypergraph for $|\mathcal{E}| \leq 3$, and for any values of n.

Proof.

As $\mathcal{H}$ is d- uniform hyperstar with n vertices, each hyperedge contains d vertices. In which there exists atleast a vertex in $|\mathcal{E}|$ hyperedges. Hence the only possible way for $\mathcal{H}$ to be relatively prime edge labeled hypergraph is $|\mathcal{E}| \leq 3$

Applications of Hypergraph

Hypergraphs are everywhere, including physics, chemistry, biology, sociology, technology, neuroscience, and ecology. In this section, some applications of hypergraphs in various fields are discussed. In chemistry, consider the elements of a complex molecule which are atoms, the large spheres being carbon atoms and the small white spheres being hydrogen atoms. Bonds/edges in complex molecules are chemical bonds between atoms.

In neuroscience, the elements of the brain include cells, such as the neurons that perform thinking and the glia that provide support. Connections in the brain are dendrites and axons that communicate electrical impulses between neurons.

In ecology, the components of an aquatic food web are collections of marine species. For instance, the letters FIS stand for fish, cep for cephalopods like squid and octopus, and cru for crustaceans like crabs, lobsters, krill, and barnacles. Connections in a pelagic food web demonstrate which marine creatures eat which other creatures.

In social media, the elements are the members of LinkedIn, and the connections in the social graph which people have connected to which other people are on LinkedIn.

Finally, the components of the Internet map are IP addresses, and if you're on the Internet right now, you have an IP address for your connection, whether it's your home, school, library, workplace, or the coffee shop you're at. Links/edges in a web map are definitely links between IP addresses.

From the above applications of hypergraphs, any node in these hypergraphs is an atom, an enzyme, a cell, a type of marine organism, a person, an internet connection, a type of thing, or something discrete, but each of the nodes has some connections between those discrete things. As a result, an edge in any of these hypergraphs represents a connection between things in the real world.

The point here is that we humans have a tendency to divide the world into discrete things and classes and concepts and name them. When we look at the creatures in the ocean, we divide that into classes like crustaceans, cephalopods, fish, and so on. When looking at the internet and we divide it into connections and number those connections as 145.251.33.68 or 198.35.26.96, each following some pattern.

As per the above discussion, it is possible to divide the world into discrete things, classes, or concepts, and it is no surprise that everything is nodes and everything else is edges. If there are nodes, then there are edges connecting them, and if everything is nodes and edges, then everything is a hypergraph. That is, a hypergraph is made up of nodes and edges of things, classes, or concepts, as well as connections between those things, classes, or concepts. As a result, hypergraphs can be found everywhere.

A hypergraph is also said to be a simple graph in which each edge contributes exactly 2 vertices. Hence, it is clear that a simple graph is a special kind of hypergraph. So, discussing examples of hypergraphs includes simple graphs too.

Application of Relatively Prime Edge Labeling Of Hypergraph

Without reliable server backups, data loss is common in business. Situations that can lead to data loss include files being accidentally deleted, servers crashing, machines malfunctioning, and people making mistakes. To deal with the above situation, it is essential to perform full backups properly and store them safely. As the most time-consuming backups are full backups, it is necessary to plan full backups accordingly. This means that full backups can be run overnight or during off-peak hours or without overlapping. During a full backup, the entire server is copied. An incremental backup backs up only data that has been modified or added since the previous backup to save storage space. Scheduling backups and disappearing isn't enough. They need to be monitored to ensure that they happen on time.

The role of backup server plays an important role in the IT environment where every system is networked through one or more servers. A backup server has a fixed hardware server with a large storage capacity and a specifically designed backup server program. Each computer's backup schedule can be set either in the host operating system (OS) or with the client application tool. The host connects to the backup server at the appointed time to start the data backup procedure. Data backup can be used in case of data loss, data corruption, or any disaster.

A backup server is a particular kind of server that provides the backup of information, including files, applications, and databases. It combines software and hardware technologies. Copying data from one source to an alternate location to protect against any disaster, accident, or malicious activity is called data backup. Modern organizations rely heavily on data, and losing such data can seriously harm and interfere with daily operations. Because of this, data backups are essential for all kinds of businesses, big and small.

With the help of relatively prime edge labeling of a hypergraph, the data backup process is scheduled to optimize the system speed. Normally, backup is done while the system is idle, but it's not possible in many IT companies, where work has been divided into both day and night times.

Consider an organization that consists of many servers and with each server, many computers are associated. Some computers share data between two servers. Hence, by considering this situation a hypergraph can be constructed, in which each server acts as an edge with many computers associated with the server as vertices. Some computers share the data from many servers. Now, the aim is to schedule the server for the backup slots.

Let us now convert the situation into a hypergraph. For example, if there are 5 servers in an organization with many computers associated with each server. Also, some computer shares two or more servers. Here, the edges in the hypergraph $\mathcal{H} = (\mathrm{V}, \mathcal{E})$ are $\{e_1, e_2, e_3, e_4, e_5\}$ where, $e_1 = \{v_1, v_2, v_3\}$, $e_2 = \{v_3, v_4, v_7\}$, $e_3 = \{v_5, v_6, v_2\}$, $e_4 = \{v_3, v_4, v_5, v_6\}$, $e_5 = \{v_1, v_4, v_7\}$.

We now label the edges of the hypergraph in a way that, the adjacent edges are relatively prime, that is, $\mathrm{L}(\mathrm{e}_1) = 3, \mathrm{L}(\mathrm{e}_2) = 2, \mathrm{L}(\mathrm{e}_3) = 4, \mathrm{L}(\mathrm{e}_4) = 1, \mathrm{L}(\mathrm{e}_5) = 5$.

Here, the label represents the slots or the days in which the backup needs to be happened. That is, server e_4, backup the data every day, Server e_1 should back up the data once in every 3 days, and so on. The labeling mentioned above gives the optimum schedule since the servers with relatively prime pairs reduce the frequency of backups happening at the same time.

Results and Discussion

Hypergraphs generalize graphs by allowing edges to connect more than two nodes (17). Hypergraphs are a versatile and inclusive model, applicable to various scenarios where entities and relationships exist. This aligns with the idea that hypergraphs can be used to represent complex relationships in diverse domains such as biology, social networks, computer science, knowledge representation, and more (18). This study presents a novel labeling method for hypergraphs, namely relatively prime labeling and relatively prime edge labeling of hypergraphs. Also, the graph that admits the above labeling is discussed. The study concludes by discussing a novel application of the labeling method within the context of hypergraphs.

Conclusion

This application could provide insights into how the labeling scheme contributes to solving specific problems, optimizing certain processes, or enhancing the understanding of relationships within hypergraphs.

Abbreviations

Nil

Acknowledgment

Nil

Authos contribution

Both the authors have equally contributed

Conflict of interest

The authors declare no conflict of interest

Ethical approval

Not applicable

Funding

This research received no external funding

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