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Unified Nash Equilibrium Model for Water Management Strategies in Smart Cities

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Abstract

Water scarcity and allocation disputes have emerged as major challenges in increasingly urbanizing smart cities, where increasing population density, outdated infrastructure, high water losses, and unequal geographic distribution frequently result in shortages despite adequate overall supply. Traditional techniques, such as linear programming and agent-based modeling, have produced helpful insights, but they are still restricted in capturing varied stakeholder behaviors, assuring equilibrium stability in competitive contexts, and providing spatially adaptable solutions. To address these shortcomings, this study applies the concept of the Nash Equilibrium (NE) model within Game theory (GT) to model strategic interactions among households, industries, utilities, and regulators, each with distinct payoff functions. Once equilibrium is achieved, no stakeholder can unilaterally improve its outcome, thereby guaranteeing fairness and stability. Building on this theoretical foundation, the model integrates Optimized Multi-Objective Particle Swarm Optimization (OMOPSO) to efficiently explore Pareto-optimal trade-offs between economic, social, and environmental objectives, while Geographic Information Systems (GIS) incorporate spatial constraints to deliver geographically realistic allocation strategies. Experimental validation demonstrates that the proposed model consistently outperforms existing approaches within the framework of Multi-Objective Evolutionary Algorithms (MOEAs) in terms of convergence stability and computational efficiency. Beyond algorithmic performance, the findings highlight practical applications for tariff design, consumer incentive programs, infrastructure investment, and water-use restrictions. This study increases state-of-the-art urban water management by integrating GT, evolutionary optimization, and spatial analysis, while also providing policymakers with a strong and fair decision-support framework for sustainable resource allocation.

Keywords: Game Theory, GIS, Nash Equilibrium, OMOPSO, Smart City, Water Management.

Introduction

Water is fundamental to achieving sustainability, functioning as a vital resource across human, industrial, and ecological sectors. This significance is underscored in the United Nations' 2030 Sustainable Development Goals (SDGs) (1), which highlight the imperative to "ensure the availability and sustainable management of water and sanitation for all". However, water scarcity is one of humanity's most urgent challenges nowadays, particularly in major urban centers. In Hanoi, one of Vietnam's most densely populated cities; the city's urbanization ratio reached 58.7% in 2020 and is projected to rise to 68.2% by 2030 in Figure 1 (2). Hanoi's daily tap water demand is estimated at 1,250,000 to 1,350,000 m^3, with a supply capacity of approximately 1,530,000 m^3 per day. While the city's water treatment facilities can nearly fulfill overall demand, uneven distribution leads to localized water shortages in certain areas in Hanoi (3). In addition, water loss is a critical factor contributing to water shortages in Vietnam generally and in Hanoi specifically. The water loss rate, once significantly high, has been gradually reduced to 21.5%, aligning with rates observed in other countries in the region; however, it remains above those seen in developed nations (4). Water loss is primarily driven by the absence of adequate technological solutions in numerous cases (5). Nowadays, several factors contribute to challenges related to clean water. One of the main causes is the conflict among stakeholders in the distribution and utilization of water, as illustrated in Figure 2 below, along with limitations in water planning and management. Consequently, both ordinary cities and those with high smart city indices are affected. Notable examples include London (ranked 8th), Beijing (ranked 13th), Hong Kong (China) (ranked 20th), and Melbourne

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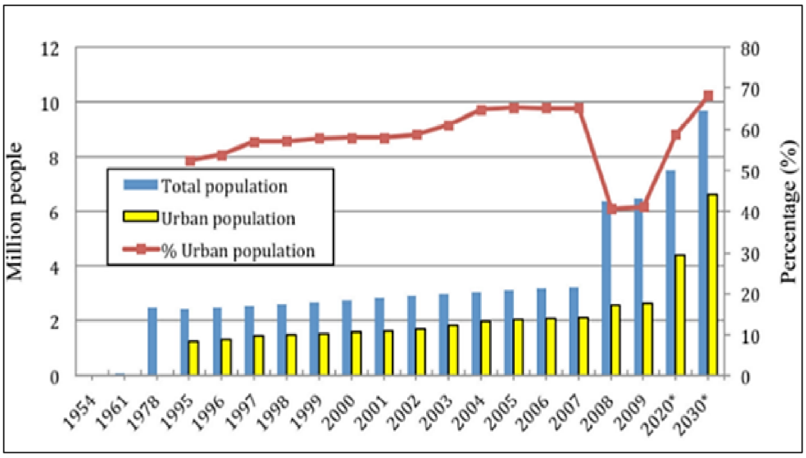

Figure 1: Hanoi's Population Growth from 1954 to 2030 (2)

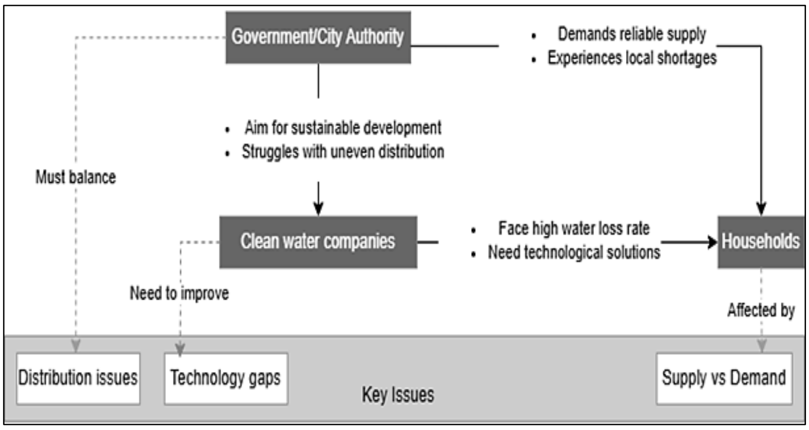

Figure 2: Stakeholder Conflicts in Smart City Water Management

(ranked 33rd) (6). To address clean water challenges in smart cities, GIS is vital, offering comprehensive tools for geospatial data acquisition, analysis, and visualization, which are crucial for infrastructure development stages (7-12). Furthermore, GT provides a framework for equitable water distribution, enabling the evaluation of sharing mechanisms to ensure fairness among stakeholders (13). Specifically, the NE is a key concept in GT, identifying optimal strategies where no player can improve their outcome by changing their strategy alone (14). The application of GT and NE aims to achieve a scenario where all stakeholders have sufficient water for economic and domestic purposes, thus enhancing the overall quality of life. Integrating GIS with GT and NE helps identify strategies that effectively balance water demands while ensuring the economic viability of water providers.

To manage complex water management, this study employed MOEAs, population-based methods effective for conflicting goals (15). MOEAs iteratively refine candidate solutions using evolutionary principles, aiming for the Pareto-optimal front, which represents the best trade-offs between objectives (16). Compared to single-objective methods, MOEAs excel in water management by simultaneously addressing economic, social, and environmental goals (17). Their ability to explore vast solution spaces makes them ideal for intricate urban water distribution planning, particularly when combined with GT and GIS (18).

In 2023, the application of GT was extended by incorporating a Unified Game-based model to effectively address water conflicts among the six countries sharing the Mekong River Basin (19). The concept of Nash Equilibrium, as a core element of Game Theory, has also been applied to solve challenges in project payment scheduling and risk management responses (20, 21). Optimal strategies to reduce disparities in water allocation within the Duck River Basin in Tennessee, USA, have been determined using NE (22). Similarly, NE has played a critical role in analyzing water resource allocation strategies for the Harirud River among Afghanistan, Iran, and Turkmenistan (23).

Beyond GT and NE, multi-objective evolutionary algorithms (MOEAs) have been widely applied to optimize water management problems. NSGA-II is recognized as an "industry-standard" and has been successfully implemented in numerous water optimization challenges (24, 25). NSGA-II was employed to address water-related issues in major cities, including New York and Hanoi (26). The PSO algorithm has likewise been utilized to tackle water-related challenges in Vietnam and China (27, 28). In addition to MOEAs, Geographic Information Systems (GIS) have been applied in wastewater systems, water distribution, energy infrastructure, and transportation networks (29). GIS-based spatial statistical methods were used to address transportation issues in Iran (30), and GIS modeling was applied to optimize land-use planning in the Erhai Lake Basin (31-35). A specialized GIS system for urban water management in Spain was introduced in 2020 (36). This enhances network optimization, improves leakage detection, and supports long-term infrastructure planning, ultimately strengthening resilience against climate change (37, 38).

Despite significant advancements in applying innovative technologies and mathematical methods to water management, critical research gaps still need to be addressed. Most existing studies focus on water management at the river basin or national level, while the rapidly urbanizing context of smart cities remains underexplored. Furthermore, social factors like stakeholder interaction and collaboration are frequently overlooked or overly simplified in current models.

Water management in smart cities presents complex challenges due to conflicting interests among stakeholders such as households, suppliers, and policymakers. These conflicts often led to inefficiency, particularly when compounded by outdated infrastructure and uneven distribution, resulting in localized shortages and significant water losses. Additionally, long-term sustainability was increasingly threatened by rapid urbanization and climate change, highlighting the need for adaptive, data-driven management approaches. To address these issues, this study proposed a novel framework that integrated NE, MOEA and GIS. NE, grounded in GT, was applied to model stakeholder interactions and to establish stable water allocation strategies in which no participant could unilaterally improve their outcome, thereby promoting fairness and cooperation. To resolve trade-offs among economic efficiency, environmental sustainability, and social equity, the study incorporated the OMOPSO algorithm, which identified Pareto-optimal solutions across multiple conflicting objectives. GIS was further embedded into the optimization process to incorporate spatial constraints and enable real-time, location-aware adjustments in water distribution, which improved infrastructure planning, leakage detection, and climate resilience. Overall, the proposed approach advanced the state of the art by defining essential optimization parameters, including infrastructure conditions, pressure levels, leakage rates, and consumer prioritization, by developing a domain-specific, game-theoretic framework for fair and efficient urban water management under real-world spatial constraints.

Methodology

Potable water management in smart cities is a difficult subject that is heavily impacted by competing interests and limited resources. Effective Smart City Water Resources Management (SCWRA) necessitates a framework for analyzing strategic interactions and negotiating dynamics among water stakeholders. Thus, we represent our problem as follows: Assume we have N participants ($N \in N^*, n > 2$) represented as Node (each $node\ i \in N^*$) on GIS, where each Node corresponds to a consumer (household, business, *etc.*) relying on clean water. Each Node is defined by coordinates (x^i; y^i) on the GIS map, representing its precise location and influencing factors such as demand, water supply access, and distribution constraints.

Each Node has a specific water demand, geographic coordinates, and priority level. The dataset in the Table 1 shows the water demand (m^3/day), coordinates(x; y), and priority level for each Node:

Table 1: Simulation Data for Water Management in Thanh Xuan District, Hanoi, Vietnam

Node	x	y	Water Demand (m^3/day)	Priority Level
1	10	20	500	1
2	15	25	2000	0.8
3	30	35	5000	0.5

In this example, the goal is to allocate water efficiently to each Node, ensuring that each receives an amount proportional to its demand while adhering to supply and distribution constraints. The simulation output in Figure 3 illustrates the management and highlights water shortages. Colors indicate the priority for redistribution: red denotes severe shortage requiring immediate action, orange indicates high priority, yellow indicates moderate priority, and green indicates low priority; gray areas reflect adequate supply with no intervention needed. Greater color intensity corresponds to greater local severity within each class.

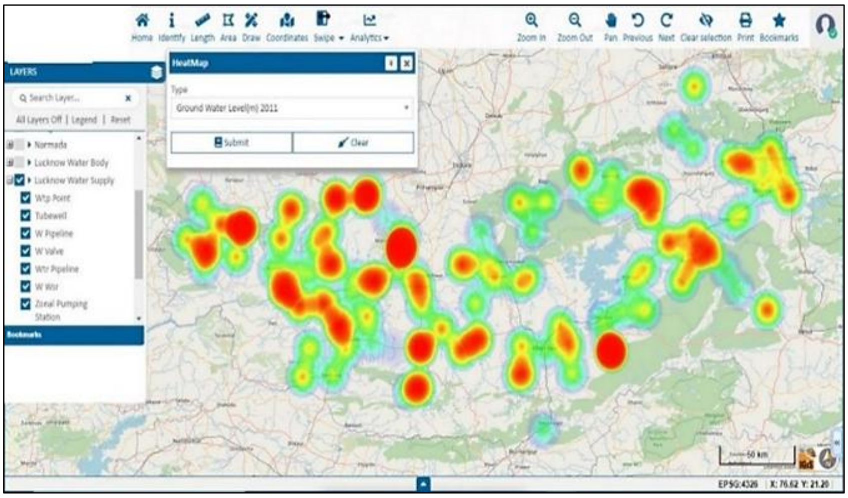

Figure 3: Water Management for Agriculture on GIS System (39)

In the SCWRA problem, each Node is characterized by attributes such as water management before leakage WA^i, flow rate FR^i, water pressure WP^i, and the optimal water pressure WP^{opt^i} ,required for efficient distribution. These characteristics are common in existing models of resource management as they are essential for determining the effectiveness of water distribution systems (40). The water demand for each Node is determined by D^i, and its leakage rate LC^i, is also a significant factor in the model (41). In addition to these established factors, each Node's strategy includes its allocated water (x^i; y^i), priority level PL^i, and leakage control strategy LC^i, ensuring equitable distribution. The main characteristics of players and their corresponding strategies used in the model are summarized in Table 2.

The SCWRA problem requires optimizing multiple objectives, including ensuring sufficient Water Management WA^i based on Water Demand D^i , minimizing water losses through Leakage Control LC^i and Pipe Flow Rate FR^i adjustments, and maintaining Water Pressure WP^i stability.

Table 2: Summarize the Characteristics of Players and Strategies

Characteristics of Players	Characteristics of Strategies
Water Management WA^i, Flow Rate FR^i, Water Pressure WP^i, Optimal Water Pressure WP^{opt^i} , Water Demand D^i	Coordinates (x^i; y^i) , Priority Level PL^i, Leakage Control rate LC^i

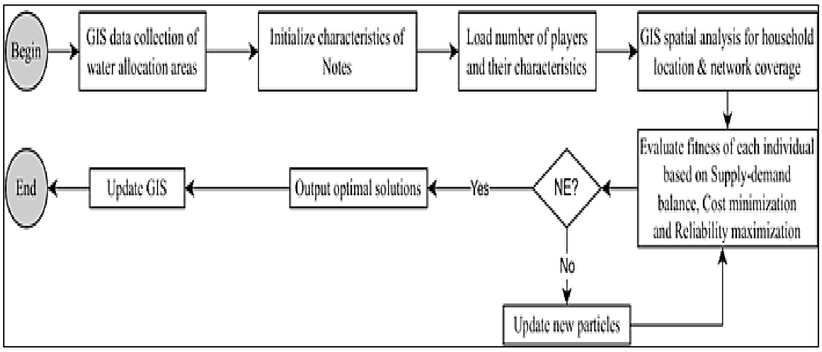

Figure 4: Flowchart for Solving the SCWRA Issue

Additionally, it involves prioritizing nodes according to their Priority Level PL^i and leakage control, aims to optimize water distribution, minimize leakage, and ensure fairness. These objectives are interdependent and must be balanced for efficient, fair, and stable water distribution, as shown in the Figure 4. The Unified Game-based model was applied in project management, conflict resolution, and multi-round auction optimization (42), providing a robust framework for modeling strategic interactions among stakeholders. Specifically, it was successfully used to model multi-agent conflicts in project management scenarios and to optimize bidding strategies in complex auction environments. This model effectively captures equilibrium conditions in cooperative scenarios with non-zero-sum and imperfect information, making it particularly suitable for addressing SCWRA problems. By leveraging Nash equilibrium concepts, we can establish an interaction framework where water suppliers and consumers strategically balance individual objectives and mutual constraints to achieve stable, optimal resource allocation.

To find a balanced solution to the problem, we proposed a novel model called the Unified Nash equilibrium model, which aims to achieve a win-win outcome in water resource allocation. For the SCWRA problem, we refer to this model as G:

$$G = \langle \{P^i, P^0\}, \{S^i, S^0\}, \{U^i, U^0\}, R^C \rangle \quad [1]$$

Where P^i=$\{p_1^i, \dots, p_M^i\}$ and $(1 < i < M)$: the array of $Note\ i$, $M \in N^*$ is the number of Nodes

- P^0: Special player (customer satisfaction), represents the overall consumer experience, capturing the impact of water management, pricing, and service quality. It ensures fairness, service efficiency, and responsiveness, balancing individual needs with system optimization. $\{S_1^i, \dots, S_K^i\}$
- S^i = $\{s_1^i, \dots, s_K^i\}$ $(1 < i < K \in N*)$ and S^0=$\{s_1^0, \dots, s_K^0\}$ $(1 < i < K)$ are set of strategies of the player P^i and P^0, respectively, $K \in N^*$ is the number of strategies
- u^i and u^0 : $S^i$and $S^0 \rightarrow \mathbb{R}$ are payoff functions for players P^i and P^0.
- R^C: a vector space representing the conflict structure in the SCWRA game. Where a nonempty vector, $v \in R^C$ captures conflicts involving N players $(N \in N*)$ in the normal form of the game. Specially, v can be expressed as a vector of strategies $(s_{pq}, \dots, s_{xy})$, with $s_{pq}, s_{xy} \in S^i$, where $(1 \leq p, x \leq N)$ and$1 \leq g \leq M_p, 1 \leq y \leq M_x$.

The main parameters of the model include: the characteristics of the players $Node\ i$ and the characteristics of the strategies S^i are defined as follows and have been outlined earlier in the problem definition. Each player is represented by the following parameters: $P^i = \{WA^i, FR^i, WP^i, WP^{opt^i}\}$ and S^i=$\{D^i, (x^i, y^i), PL^i, LC^i\}$.

In the proposed model, convergence to a unique Nash equilibrium is guaranteed when the following conditions are satisfied:

(i) each player P^i (including the special player P^0) has a non-empty, compact, and convex strategy set $S^i \subset \mathbb{R}^K$; (ii) the payoff functions U^i: $S^i \times S^{-i} \rightarrow \mathbb{R}$ are continuous in all strategies and quasi-concave in the player's own strategy; and (iii) the conflict structure vector space R^C ensures that no strictly dominated strategies exist, thereby eliminating cycles of best responses. Under these conditions, existence and uniqueness follow from standard fixed-point theorems (Debreu-Fan-Glicksberg).

In practice, uniqueness is confirmed by implementing an iterative best-response algorithm, where each player updates its strategy $s^i \in S^i$ to maximize U^i given the strategies of others. Convergence is declared once $max_i \parallel s_{t+1}^i - s_t^i \parallel < \epsilon$, with ϵ denoting a small tolerance threshold. Numerical experiments on the SCWRA dataset consistently showed convergence within finite iterations, and sensitivity analysis confirmed that the equilibrium solution was robust to variations in initial strategy profiles. The presence of the special player P^0, representing consumer satisfaction, further stabilizes the system by penalizing deviations that reduce global efficiency, which strengthens uniqueness compared to classical Nash formulations.

Based on the characteristics of the proposed model, the payoff functions are designed as follows:

Payoff for Each Node *i*: The actual amount of water provided to each Node

$$u^i = WA^i - \left(\frac{24}{FR^i} . PL^i . \sqrt{(x^i - x^j)^2 + (y^i - y^j)^2}\right) - (LC^i . WA^i) - \left(\frac{|WP^i - WP^{opt^i}|}{WP^{opt^i}} . WA^i\right) \quad [2]$$

Payoff for Special player:

$$u^0 = \sum_{i=1}^{M} WA^i - \sum_{i=1}^{M}(D^i - WA^i) - \sum_{i=1}^{M} \left| \frac{WA^i}{D^i} - \frac{1}{M}\sum_{j=1}^{M} \frac{WA^j}{D^j} \right| \quad [3]$$

where: $\frac{24}{FR^i}$: Time to supply per day (m^3/day)

$(x^i - x^j)^2 + (y^i - y^j)^2$: the distance from Node i to source (km)

$LC^i . WA^i$: Leakage Loss (m^3/day)

$\frac{|WP^i - WP^{opt^i}|}{WP^{opt^i}} . WA^i$: Leakage loss based on pressure loss (m^3/day)

Our model effectively addressed the SCWRA problem by integrating players' strategies, requiring that no player can unilaterally deviate to improve their payoff: $u^i(s_i^*, s_{-i}^*) \geq u^i(s_i, s_{-i}^*) \forall s_i \in S^i, \forall i \in \{1, \dots, M\}$. It balances system-wide objectives with individual player preferences, ensuring fair resource distribution. The model's computational efficiency comes from its clear mathematical relationships, which simplify calculation and optimization. The model's output may be used by local governments and water organizations to guide several decision-making levels. In particular, the model's equilibrium techniques show the best allocation patterns that strike a compromise between infrastructure limitations, stakeholder demands, and environmental goals. Policymakers should use these results as a guide for creating fair and effective water distribution plans that minimize conflict among stakeholders, maximize resource utilization, and enhance compliance. Through GIS integration, the model also offers spatially resolved insights, allowing for targeted actions based on regional differences in demand and supply.

Water management in smart cities is an NP-hard problem, indicating that its solution space grows exponentially, rendering optimal solutions computationally infeasible for large-scale systems (43). Specifically, for n nodes $(n \in N*, n > 2)$, the number of potential coalitions among nodes reaches 2^n , rendering precise computation infeasible due to the vast solution space. This complexity is heightened by conflicting goals, including optimizing water management WA^i , minimizing losses $LC^i . WA^i$, maintaining optimal pressure WP^i, and ensuring fairness u^0 , with spatial distances $(x^i - x^j)^2 + (y^i - y^j)^2$ requiring GIS data.

OMOPSO models Node as a swarm of particles for SCWRA $G = \langle \{P^i, P^0\}, \{S^i, S^0\}, \{U^i, U^0\}, R^C \rangle$ where each particle encodes Strategies s^i using parameters $PR^i . LC^i$ and WA^i, aiming to achieve solutions aligned with NE principles. It tackles NP-hard complexity (2^n solution spaces) by iteratively updating particles via parallel processing, guided by p_{best} and g_{best} , with velocity constraints balancing exploration and exploitation. Mutation ensures diversity, while GIS data (x^i, y^i) adjusts FR^i, yielding Pareto-optimal solutions for costs, water losses, and pressure stability.

To implement OMOPSO for SCWRA, each particle represents a chromosome encoding a water distribution plan, with components defining parameters, such as flow quantities FR^i, leakage mitigation factors LC^i, and pressure settings WP^i across all Node in Thanh Xuan District. The particle integrates Node attributes like demand D^i, priority PL^i , and geographic positions (x^i, y^i). Specifically, a particle is defined as $X = [FR^i, LC_1{}^i, WP_1{}^i, \dots, FR_M{}^i, LC_M{}^i, WP_M{}^i]$ where M is the number of *Nodes*, and subscripts denote *Nodes* indices within the array for each $Node\ i$. An illustrative particle as Figure 5 is proposed as X = [100, 0.1, 4, 200, 0.15, 4.5, 300, 0.2, 5].

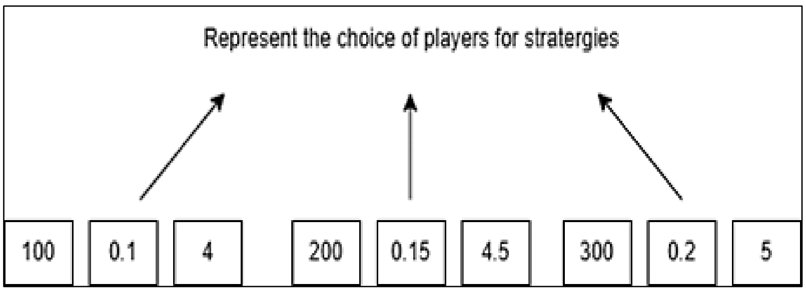

Figure 5: A sample Chromosome

Where each triplet corresponds to $[FR^i, LC_1{}^i, WP_1{}^i]$ for Node 1, Node 2, and Node 3, respectively. The fitness function minimizes differences between Node by striking a balance between efficiency and justice while assessing the quality of a water management plan. When $|u^i - u^j|$ approaches 0, indicating near-equal benefits among Nodes, Fitness decreases, signifying an optimal solution; conversely, as $u^i - u^j$ increases, the value rises, reflecting greater inequality and less effective management.

The fitness function is defined as follows:

$$F = \frac{\sum_{ij}^{M} |u^i - u^j|}{2 \times M^2 \times \frac{u^0}{M}} \quad [4]$$

Where:

- u^i the payoff function for a normal player $Node\ i$
- u^j is the payoff function for a normal player $Node\ j$
- u^0 is the payoff function for a special player
- M ($M \geq 1$) is the number of water management $Notes$ in the system.

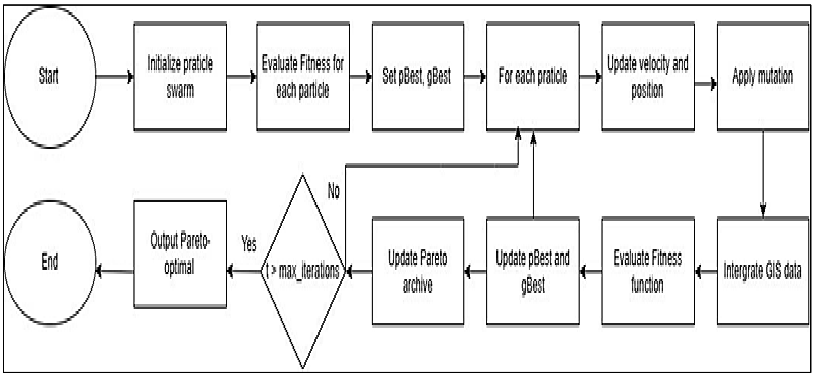

Figure 6: Flowchart of the OMOPSO Algorithm for the Problem

The OMOPSO algorithm initializes a swarm of particles, each encoding a water distribution plan for Thanh Xuan District Nodes, using GIS data (x^i, y^i) It iteratively refines these plans by updating positions, applying mutation, and evaluating fitness to ensure fairness and efficiency. The process repeats until a Pareto-optimal set is delivered, balancing management, leakage, and pressure stability. Based on the workflow delineated in Figure 6, the pseudo code for OMOPSO (Figure 7) formalizes its methodology for addressing the SCWRA problem. It initializes a swarm of particle encoding strategies s^i, evaluates fitness using GIS coordinates (x^i, y^i), and iterates over T cycles with mutation to refine positions and archive non-dominated solutions in A. The algorithm terminates upon reaching the iteration threshold, yielding a Pareto-optimal set of water management strategies.

Algorithm 1 OMOPSO Algorithm for SCWRA

Require: Notes data $(D^i, PL^i, (x^i, y^i))$, Constraints (WA^i, LC^i, WP^i), Swarm size N, Max iterations T, $V_{\max}$, Mutation rate.

Ensure: Pareto archive A of optimal water allocation plans.

1: Initialize swarm S with N particles $X = [FR_1^i, LC_1^i, WP_1^i, \ldots, WP_1^M]$ for M nodes and velocities V within constraints; set Pareto archive $A = \emptyset$.

2: Evaluate fitness $F(X)$ for each particle $X \in S$ using GIS data $((x^i, y^i))$ to compute u^i; set $p_{best} = X$ and g_{best} as the best particle in S.

3: **for** $t = 1$ to T **do**

4: **for** each particle $X \in S$ **do**

5: Update velocity $V = w \cdot V + c_1 \cdot rand() \cdot (p_{best} - X) + c_2 \cdot rand() \cdot (g_{best} - X)$, then position $X = X + V$; integrate GIS data (x^i, y^i) to adjust WA^i.

6: Apply mutation to X with probability $mutation\_rate$, evaluate fitness $F(X)$, and update $p_{best} = X$ if $F(X) > F(p_{best})$.

7: **end for**

8: Update g_{best} as the best particle in S; update Pareto archive A with non-dominated solutions from S.

9: **end for**

10: **return** A.

Figure 7: Pseudo Code of OMOPSO Algorithm in SCWRA

In experiment, all computational tests were executed on a high-performance system configured with a 12th Gen Intel(R) Core(TM) i5-12450H processor, an NVIDIA GeForce GTX 1650 GPU, and 15.7 DDR4 RAM. These hardware specifications were chosen to optimize the algorithm's performance. Table 3 below details the experimental parameters used in this study.

Table 3: Experiment Parameters for OMOPSO

Swarm Size(N)	**Maximum Iterations (T)**	**Inertia Weight (w)**	**Cognitive Coefficient (c^1)**	**Social Coefficient (c^2)**	**Mutation Probability**	**Speed Constraints ($Vmax$)**
50	100	0.5	2.0	2.0	0.1	0.6

Results and Discussion

This study introduces a dataset with weighted values for factors affecting water management across four Nodes, each assessed with two strategies. Key indicators for each Node include allocated water, distance from the water source, priority level, leakage coefficient (scaled by a factor of 10 for optimization consistency), water demand, water pressure, and optimal pressure, as summarized in the table below for the first four Nodes. Utilizing the dataset partially outlined in Table 4, the experimental outcomes are summarized in Table 5, Table 6, and Figure 8. Table 5 displays the optimal water management solution for each Node, including its respective payoff value, highlighting the balance between gains and losses across the Node through the application of the OMOPSO algorithm.

Leveraging the optimal strategies and corresponding payoff values presented in Table 5, Table 6 evaluates the performance of various optimization algorithms (VEGA, NSGAII, NSGAIII, SMPSO, PESA2, and OMOPSO) when applied to the dataset. The figure quantitatively assesses the convergence efficiency of each algorithm toward the optimal solutions derived from Table 5, illustrating their fitness values across multiple iterative cycles.

Table 4: A part of the Dataset for Nodes

ID	Strategy	WA^i	Distance	PL^i	LC^i	DL^i	WP^i	WP^{opy^i}
1	1	1000	0.36	0.07	0.08	4814.1	2.39	8.87
	2	2000	0.74	0.05	0.06	2188.5	1.09	6.38
2	1	1000	0.21	0.07	0.02	4084.9	5.82	6.16
	2	2000	0.62	0.03	0.02	7245.5	5.97	9.24
3	1	3000	0.86	0.07	0.02	9411.5	2.36	9.87
	2	4000	0.29	0.08	0.05	3466.8	3.81	6.87
4	1	1000	0.33	0.04	0.01	2140.7	2.42	8.17
	2	2000	0.22	0.02	0.08	6344.9	4.13	9.69

Table 5: Result of the Experiment

Player	Chosen Strategy Name	Payoff Value
Node 1	Strategy 38	6230.93
Node 2	Strategy 25	9510.88
Node 3	Strategy 5	6988.82
Node 4	Strategy 53	3717.33
Node 5	Strategy 8	9162.32

Table 6: Comparison of Fitness Value of Different Algorithms

Inter	VEGA	NSGAII	NSGAIII	SMPSO	PESA2	OMOPSO
1	221.48	0.93	1.69	10-3	29.29	13.99
2	359.74	0.11	0.70	0.04	7.70	0.02
3	535.91	7.51	2.60	910-4	0.26	0.25
4	92.39	1.70	0.04	0.03	149.27	1.65
5	268.16	0.09	23.99	0.65	0.07	0.01
6	8.66	16.37	2.98	0.02	5.61	0.03

7	783.54	0.32	0.61	2.04	61.80	10-3
8	505.06	2.26	1.62	3.47	1.69	15.49
9	2727.70	2.51	22.74	0.07	38.23	0.40
10	543.47	5.10	0.43	0.97	32.12	0.03

Table 6 examines the fitness values of six optimization algorithms (VEGA, NSGAII, NSGAIII, SMPSO, PESA2, and OMOPSO) across multiple iterations. The table reports the fitness value for each algorithm, which indicates the quality of the solution (scaled by a factor of 100,000 for presentation); for instance, OMOPSO at the first iteration records a fitness value of 13.99. These metrics facilitate a quantitative assessment of each algorithm's convergence performance in deriving optimal solutions.

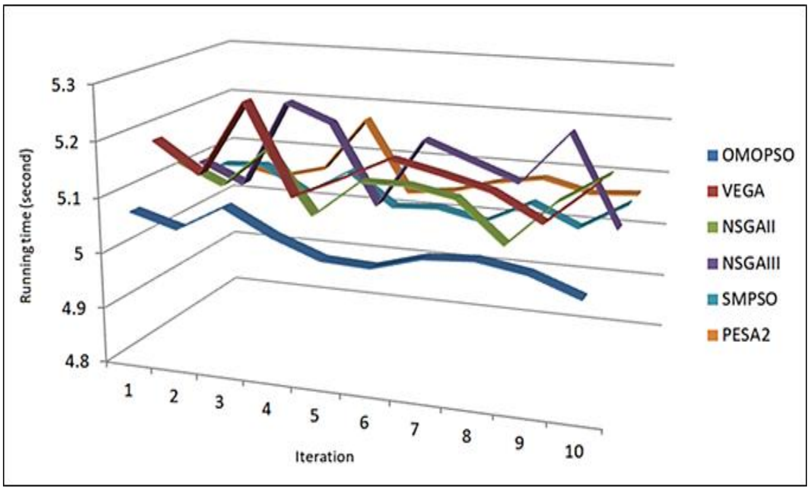

Figure 8: Comparison of Runtime (In Seconds) of Different Algorithms

Figure 8 depicts a runtime comparison of multiple optimization algorithms (VEGA, NSGAII, NSGAIII, SMPSO, PESA2, and OMOPSO) via a line chart, detailing the computational duration of each algorithm across successive iterations. Collectively, Table 7 and Figure 8 offer a rigorous assessment of the algorithms' computational efficiency and speed, enabling the determination of the optimal algorithm for solving the given problem.

Table 6 and Figure 8 compare the fitness values and runtimes of the VEGA, NSGAII, NSGAIII, SMPSO, PESA2, and OMOPSO algorithms across ten trials for the multi-objective optimization problem of SCWRA. In Table 7, OMOPSO demonstrates superior performance with fitness values ranging from 0.001 to 15.49, achieving the lowest at the 7th iteration, far surpassing VEGA with an unstable range from 8.66 to 2727.70, peaking at the 9th iteration. NSGAIII maintains stability from 0.04 to 23.99, highest at the 5th iteration, yet lags behind OMOPSO, while NSGAII from 0.09 to 16.37, SMPSO from 0.0009 to 3.47, spiking at the 8th iteration, and PESA2 from 0.07 to 149.27 exhibit inconsistent performance.

Figure 8 highlights OMOPSO's computational efficiency with a stable runtime of 5.001 to 5.096 seconds, ranking among the fastest. NSGAIII peaks at 5.229 seconds in the 3rd iteration, showing higher cost, while VEGA varies from 5.094 to 5.263 seconds with moderate efficiency, and NSGAII (5.024-5.163 seconds) and SMPSO (5.012-5.084 seconds) remain consistent. OMOPSO's low, stable runtime and competitive fitness affirm its reliability for SCWRA optimization.

Figure 9 illustrates the spatial distribution of water management results in Thanh Xuan District on a GIS map, utilizing optimal solutions derived from OMOPSO. It marks the locations of 10 Nodes, color-coded to represent the allocated water. This visualization demonstrates the framework's ability to achieve equitable and efficient water distribution across the district.

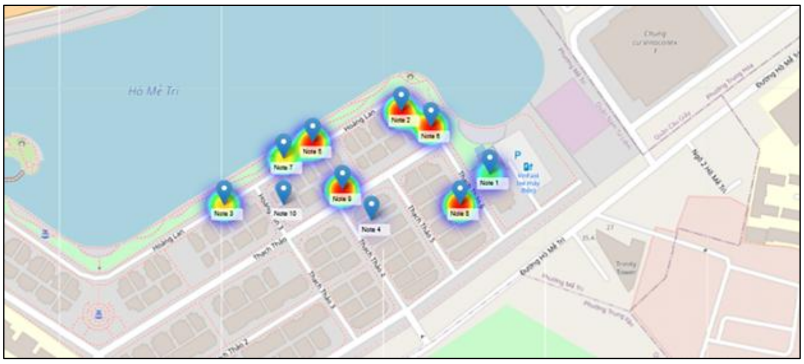

Figure 9: Spatial Distribution of Water Allocation in Thanh Xuan District, Ha Noi, Vietnam

The experimental results provided a quantitative validation of OMOPSO's superior efficacy in addressing the multi-objective water management problem (SCWRA). OMOPSO consistently achieved the lowest fitness values (0.001-15.49), outperforming VEGA's unstable range (8.66-2727.70), while NSGAIII (0.04-23.99), NSGAII (0.09-16.37), PESA2 (0.07-149.27), and SMPSO (peak 3.47) fell short of its precision and consistency, emphasizing OMOPSO's superior convergence in complex multi-objective scenarios. Additionally, OMOPSO's stable runtime (5.001-5.096 seconds) surpassed NSGAIII (peak 5.229 seconds), VEGA (5.094-5.263 seconds), NSGAII (5.024-5.163 seconds), and SMPSO (5.012-5.084 seconds), confirming its efficiency and rapid convergence, which is vital for timely water management decisions.

Beyond direct optimization performance, our study advanced existing research on MOEAs and OMOPSO applications by integrating these methodologies with theoretical constructs such as GT and Nash Equilibrium. Additionally, the comparative fitness analysis suggested the potential integration of GIS to enhance smart city applications by embedding socio-dynamic factors into optimization models. Such an interdisciplinary approach strengthens the theoretical foundation for the proposed methodology, offering new perspectives on sustainable urban water management.

Conclusion

This study proposed an integrated approach combining GIS with a NE-based model to address the SCWRA problem. The NE model provides a robust mathematical structure for modeling strategic interactions among suppliers, consumers, and system performance entities. By explicitly formulating players' strategies and payoffs and solving for Nash Equilibrium, the model captures stable states where no participant can improve their outcome by unilaterally changing their strategy. The conflict structure is represented through a vector space R^c, ensuring detailed modeling of strategic tensions across the system. This combination of GIS and NE modeling offers a powerful decision-support framework, balancing individual interests and system-wide objectives in complex water resource allocation scenarios.

Despite these contributions, the study was subject to several constraints. The NE formulation assumed rational behavior and complete information, which may not fully reflect real-world stakeholder decision-making. Furthermore, the analysis was conducted under static equilibrium conditions, without explicitly incorporating dynamic changes in demand, rainfall variability, or long-term climate uncertainty. The model also did not consider potential cooperative agreements or coalition-building mechanisms, which may influence allocation outcomes in practice.

Future research should extend the framework by integrating dynamic or repeated game formulations to capture time-dependent adjustments in stakeholder strategies. Incorporating stochastic variables related to hydrological uncertainty and climate change scenarios would further improve robustness. In addition, combining the NE model with cooperative game theory or agent-based modeling could offer deeper insights into coalition formation and adaptive management. Finally, applying the framework to real-world case studies with empirical data would help validate its practical applicability and inform policy design.

Abbreviations

SCWRA - Smart City Water Resource Allocation,

NE – Nash Equilibrium,

MOEA- Multi-Objective Evolutionary Algorithm,

GIS – Geographic Information System,

GT - Game theory,

OMOPSO – Optimized Multi-Objective Particle Swarm Optimization.

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Author Contributions

Trinh Bao Ngoc: contributed primarily to the preparation and writing of the manuscript, Do Thi Ngoc Anh: participated in developing the research idea, discussing the model jointly with Nguyen Minh Hieu, and carried out the study under the supervision of Trinh Bao Ngoc, Vu Huu Thong and To Thanh Thai: contributed to dataset collection and field validation, Hoang Phuong Thao, Le Thi Chung, and Pham Thi Tuyet: provided financial support that enabled the successful completion of this research.

Conflict of Interest

The authors declare no conflict of interest.

Declaration of Artificial Intelligence (AI) Assistance

We confirm that no Artificial Intelligence (AI) tools were used in the preparation or writing of this manuscript.

Ethics Approval

Not Applicable.

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