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Construction and Psychometric Properties Structure and Measurement of Mathematical Reciprocity: Exploratory and Confirmatory Factor Analysis

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Abstract

This study develops and validates an Indonesian mathematical resilience scale for elementary students through robust psychometric testing. Addressing the absence of culturally appropriate measurement tools, the research employs a mixed-method validation design combining exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) using JASP software. Participants included 405 students (aged 10-12) from five Central Java regencies, selected through stratified random sampling. The 15-item Likert-scale questionnaire underwent principal component analysis (EFA) followed by maximum likelihood estimation (CFA), evaluating four theoretical dimensions: Value, Struggle, Growth, and Perseverance. Results demonstrated excellent model fit (CFI=0.912; RMSEA=0.047; SRMR=0.045) with strong composite reliability (CR=0.78-0.92). While three dimensions exhibited adequate convergent validity (AVE>0.50), Perseverance showed marginal acceptability (AVE=0.47), suggesting potential cultural nuances in persistence measurement. The validated instrument enables precise identification of resilience profiles, informing targeted pedagogical interventions. This study contributes methodologically by demonstrating the sequential EFA-CFA approach's effectiveness in educational instrument development, and practically by providing teachers with a diagnostic tool for fostering mathematical resilience. Findings highlight the importance of integrating affective constructs into mathematics pedagogy and support the development of resilience-based instructional practices. Future research should expand validation to other Indonesian regions and examine the scale's predictive validity for academic achievement across different instructional contexts.

Keywords: Confirmatory Factor Analysis (CFA), Exploratory Factor Analysis (EFA), Mathematical Reciprocity, Mathematical Resilience, Psychometric Validation.

Introduction

Mathematical resilience serves as a fundamental component in primary education by cultivating students' perseverance, adaptability, and problem-solving competencies (1). This conceptual framework empowers learners to engage with mathematical concepts confidently, persist through academic challenges, and actively participate in conceptual exploration (2). When encountering difficulties, resilient students demonstrate the capacity to sustain motivation, regulate emotional responses, and devise alternative problem-solving strategies. These attributes prove particularly valuable in mathematics education, where cognitive and affective processes dynamically interact. The principle of mathematical reciprocity further enriches this understanding by highlighting the mutual reinforcement between cognitive engagement and emotional resilience (3).

Mathematical reciprocity encompasses both cognitive and social processes that occur through mutual engagement in mathematical activity. Cognitively, reciprocity involves shared reasoning, bidirectional exchange of strategies, and co-monitoring of understanding, allowing learners to refine their thinking through iterative feedback. Socially, it includes collaborative meaning-making, responsiveness to peers' ideas, and the co-construction of mathematical explanations within a supportive interpersonal structure. These processes create reciprocal dynamics in which students simultaneously influence and are influenced by others' mathematical thinking, thereby strengthening conceptual understanding

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and emotional resilience. Additionally, the reciprocity paradigm fundamentally transforms conventional educational dynamics by reconfiguring teacher-student relationships, facilitating collaborative learning environments where knowledge is co-constructed rather than unidirectional imparted (4–7).

Reciprocal learning contexts have been shown to foster creative social interactions that enhance mathematical comprehension and support learning through errors, particularly within developmental psychology frameworks (8–12). Such environments significantly improve students' ability to manage cognitive demands and derive meaningful learning experiences, highlighting the pedagogical importance of cultivating classroom cultures that support both resilience and reciprocal learning processes. Despite increasing scholarly recognition of mathematical resilience, significant measurement gaps persist, particularly concerning Indonesian elementary students (13). Existing assessment instruments frequently fail to incorporate relevant cultural and educational contexts, while the majority of research focuses on secondary or tertiary education levels. Moreover, few studies have implemented comprehensive validation methodologies combining both exploratory (EFA) and confirmatory factor analysis (CFA). This limitation becomes particularly evident when examining related constructs such as self-efficacy and motivation which interact with resilience yet require distinct measurement approaches (14, 15). The current investigation addresses these methodological limitations through systematic instrument development and validation.

The importance of mathematical reciprocity lies in its dual role as both a learning mechanism and an evaluative lens. In learning, reciprocity enhances metacognitive regulation, supports productive struggle, and strengthens conceptual retention through shared reasoning. As an evaluative construct, it provides insight into how students mobilize strategies collaboratively rather than individually, offering a complementary dimension to resilience, self-efficacy, and motivation. Unlike resilience, which emphasizes persistence, or self-efficacy, which focuses on perceived competence, mathematical reciprocity highlights mutual interdependence in mathematical engagement. At the same time, these constructs interact extensively: reciprocal exchanges can increase resilience by normalizing struggle, and they can strengthen self-efficacy by allowing students to witness peer reasoning. This interconnectedness positions reciprocity as a critical component in holistic mathematics learning frameworks.

The research novelty manifests in several dimensions: the validation of a four-dimensional instrument specifically developed for Indonesian elementary students, the innovative combination of EFA and CFA analyses within a large-scale study, and the utilization of open-access JASP software to ensure analytical transparency and replicability. Future research directions may include longitudinal impact assessments and cross-cultural applications of the developed instrument. This study aims to analyse the factor structure of the mathematical resilience instrument through EFA, validate the model using CFA, and examine its psychometric properties including reliability and validity.

Mathematical Resilience in Elementary School: Concept and Development

The concept of resilience emerged in the 1970s, psychological research on mental health initially viewed as an exceptional trait enabling individuals to overcome adversity (16–18). Mathematical resilience specifically denotes perseverance and positive attitudes in mathematics despite difficulties, countering widespread math anxiety (19–21). This unique construct stems from teaching methods, mathematics' inherent characteristics, and fixed-ability beliefs (22-25). Educational resilience involves overcoming learning obstacles through attributes, processes, and outcomes, measured by positive responses to adversity (13, 26, 27). In mathematics, it enables students to endure failure while maintaining problem-solving confidence (2). Mathematical resilience enhances problem-solving skills for academic and real-life challenges while reducing negative attitudes toward math (28, 29). Its three key factors, grades, effort, and development, correlate with stronger motivation and achievement, crucial for educational progression (30-33).

The construct comprises four components: valuing mathematics' daily relevance, accepting learning struggles, believing in skill development (growth), and persistent problem-solving (perseverance). These elements form the theoretical foundation for resilience measurement in education, fostering adaptive learning behaviors and academic success. At the elementary school level, students' cognitive and affective development greatly affects attitudes toward mathematics (34). Resilience requires self-awareness and conscious effort to manage cognitive and affective responses, significantly contributing to math success (34, 35). Parents, as the first educators, play a critical role in cognitive development and educational outcomes, including mathematics (36, 37). Family background also explains resilience levels in elementary and secondary students (38). Early perceptions of academic ability, if negative, can lead to math anxiety and hinder achievement. Thus, measuring mathematical resilience early is vital to detecting issues and designing effective interventions, enabling supportive learning approaches.

Psychometric Validation of Instruments

Measurement instruments function to collect a variety of data related to demographics, knowledge, attitudes, behaviors, and various other constructs (39). Psychometric validation is an important step in ensuring that an instrument actually measures the intended construct (40). The validation process includes construct validity tests (the compatibility between theory and measurement results), discriminant validity (the ability to distinguish different constructs), and reliability tests (consistency of measurement results). In addition, testing the fit model through CFA analysis, it is also necessary to ensure that the developed factor structure is in accordance with the empirical data. In the context of tool validation, the exploratory factor analysis (EFA) procedure is used to explore the latent structure of a set of items, while confirmatory factor analysis (CFA) is used to confirm whether the hypothesized factor model matches the data. The use of these two procedures in sequence allows for the development of more accurate instruments.

Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA)

Exploratory factor analysis (EFA) is a statistical technique used to identify patterns of relationships between a number of variables without a definite initial model (41). EFAs are useful for exploring the structure of factors hidden in data and determining the number and nature of those factors. On the other hand, Confirmatory Factor Analysis (CFA) is used to test the suitability between the size of a construct and the hypothesized measurement model, based on theoretical foundations and/or previous research findings (42, 43). CFA is confirmatory, which serves to test the validity of constructs as well as assess the reliability of an instrument (44). In this study, a two-stage method with the use of EFA and CFA was applied to gain a deeper understanding and stronger validation of the structure of the mathematical resilience instrument factors. This approach not only ensures the independent exploration of the structure but also provides model confirmation based on statistical fit standards.

Methodology

Design Research

This study employs a quantitative approach with a cross-sectional research design aimed at examining the structural validity and psychometric properties of the Mathematical Resilience Scale for Elementary Students (MRS-ES). The cross-sectional design was selected because data were collected from participants at a single time point, enabling researchers to capture the status of mathematical resilience among elementary students without variable manipulation (45). Using Classical Test Theory (CTT), we employed sequential Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA) for validation, chosen for their interpretability and suitability for initial validation studies.

Participants

The study involved 405 elementary students (aged 10-12) from Central Java, Indonesia, selected via stratified random sampling to ensure representativeness. The sample size of 405 students provides strong statistical adequacy for both EFA and CFA procedures. Psychometric standards recommend a minimum of 300 participants or at least 10–20 respondents per item to ensure stable parameter estimation in structural equation modeling. With 15 items, the present sample well exceeds these thresholds. Furthermore, the stratified random sampling strategy based on district, school type, and grade level supports representativeness across the Central Java region. This sampling frame enhances the generalizability and stability of the resulting factor structure. Participants were grades 5-6 students attending mathematics classes in 2024/2025. Ethical guidelines were strictly followed, with voluntary participation, parental consent, and confidentiality ensured.

Table 1: Aspect and Indicator of Instrument

Aspect of Resilience	Indicator		Statement
Value	V1	Belief in the relevance of mathematics in everyday life.	Learning mathematics at school is beneficial for everyday life.
	V2	Perception of the importance of mathematics for academic achievement and future career success.	Success in future careers is not determined by learning mathematics.
	V3	Awareness of mathematics as a necessary skill for problem-solving.	Learning mathematics is necessary for solving problems.
	V4	Perspective that learning mathematics provides long-term benefits.	Learning mathematics does not offer long-term benefits.
Struggle	S5	Willingness to face difficulties in learning mathematics without giving up.	Easily gives up when facing difficulties in learning mathematics.
	S6	Understanding that even experts in mathematics must persist to grasp certain concepts.	Aware that even mathematics experts must persistently strive to understand certain concepts.
	S7	Awareness that mastering difficult mathematical material requires extra effort.	Does not make extra efforts to master difficult mathematical material.
Growth	G8	Belief that mathematical ability or competence can be developed through practice and effort.	Mistakes made while learning mathematics are valuable opportunities for learning.
	G9	Perspective that mistakes in mathematics are significant learning opportunities.	Mathematical ability cannot be developed through practice and effort.
	G10	Positive attitude toward opportunities to develop mathematical skills.	Achieving success in learning mathematics requires time and sustained commitment.
	G11	Understanding that success in learning mathematics demands time and continuous commitment.	Easily gives up when opportunities to learn mathematics arise.
Perseverance	P12	Willingness to keep trying to solve difficult mathematical problems despite obstacles.	Always strives to find solutions to mathematical problems.
	P13	Persistence in seeking solutions to challenges in solving mathematical problems.	Unwilling to keep trying to solve difficult mathematical problems when faced with many obstacles.
	P14	Belief that optimal results are achieved through perseverance and diligence in learning mathematics.	Achieving good grades in mathematics due to perseverance and diligence.
	P15	Attitude of not giving up when faced with failure in mathematics.	Easily gives up when faced with failure in mathematics.

Instrument and Data Collection

This study employed a specially developed questionnaire consisting of 15 items measured on a five-point Likert scale (1 = Strongly Disagree to 4 = Strongly Agree) to assess four core dimensions of mathematical resilience: Value (4 items assessing the importance of mathematics), Struggle (3 items evaluating acceptance of challenges), Growth (4 items examining beliefs about skill development), and Perseverance (4 items measuring persistence in overcoming obstacles). The measured dimensions were aligned with established indicators of mathematical resilience (46, 47).

The instrument underwent rigorous validation, including expert review by three specialists in mathematics education and psychology for content validity and incorporation of reverse-coded items to minimize bias, ultimately establishing the MRS-ES as a psychometrically sound tool for assessing mathematical resilience in both Indonesian and international educational contexts. This instrument included several reverse-coded items to reduce acquiescence bias. All reverse-coded items were recoded before the EFA and CFA procedures to ensure the correct directionality of the latent construct. Recoding procedures followed standard psychometric guidelines and were performed before reliability and validity testing. The aspects and indicators of the instrument are presented in Table 1.

Statistical Analysis Plan

The instrument's psychometric evaluation included comprehensive validity testing through EFA and CFA. Initial EFA analysis examined item correlations, with a minimum 0.30 coefficient required for inclusion (48). The Kaiser-Meyer-Olkin measure confirmed sampling adequacy (KMO >0.5), while Bartlett's test of sphericity ($p<0.05$) verified factor analysis appropriateness. Using principal component analysis, factors were extracted based on eigenvalues exceeding 1, with each factor containing at least three significantly loading items (49). The rotated solution organized items into conceptually coherent factors, as depicted in Figure 1's validation framework.

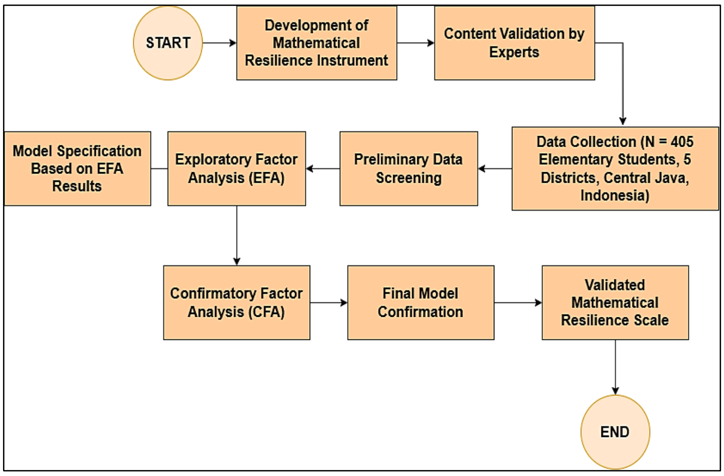

Figure 1: Factor Validation Process of the Mathematical Resilience

Figure 1 provides a visual summary of the validation procedures applied in this study, illustrating the sequential stages from item screening, EFA extraction, factor retention, and onward to CFA confirmation. This diagram clarifies how the empirical structure was systematically refined before being tested using a confirmatory model. The following EFA criteria are outlined in Table 2.

Table 2: Test for EFA

Statistical Test Used	Ideal Criteria
KMO Measure	KMO ≥ 0.50
Bartlett's Test	$p < 0.05$

Subsequently, CFA was conducted to validate the proposed factor structure. CFA is particularly useful when researchers have prior theoretical or empirical knowledge about the underlying latent variables (50). This analysis was performed using the JASP software, and the fit of the model was assessed through indices such as the root mean square error of approximation (RMSEA), standardized root mean square residual (SRMR), Tucker-Lewis index (TLI), Bentler-Bonett Normed Fit Index (NFI), and Comparative Fit Index (CFI) (51, 52). The following CFA criteria are outlined in Table 3.

Table 3: Test for CFA

Statistical Test Used	Ideal Criteria
Standardized Loadings	≥ 0.50 (ideally ≥ 0.70)
χ^2/df (Relative Chi-square)	< 3.00
CFI, TLI	≥ 0.90
RMSEA, SRMR	≤ 0.08
√AVE (Average Variance Extracted)	Each construct must be greater than the highest correlation between that construct and any other construct
CR (Composite Reliability)	CR ≥ 0.70
Cronbach's Alpha	α ≥ 0.70

The study assessed the instrument's reliability through both Cronbach's alpha (with values >0.70 indicating acceptable reliability) and composite reliability via structural equation modeling (SEM), a robust method for non-homogeneous components that yields dependable coefficients for large samples (53, 54). This comprehensive psychometric approach, aligning with current standards in high-impact journals, confirmed the instrument's strong validity (construct, convergent, and discriminant) and internal consistency, establishing its suitability for measuring mathematical resilience among Indonesian elementary students. The rigorous methodology not only ensures psychometric soundness for the current sample but also provides a robust foundation for future applications and cross-contextual studies of mathematical resilience in primary education settings.

Results

The results section provides a comprehensive overview of the data distribution, along with the outcomes of CFA and EFA. The distribution analysis highlights the variation across the research variables, illustrating central tendencies, spread, and response patterns, which are essential for understanding the data structure prior to conducting more advanced analyses. Descriptive Statistics of Mathematical Resilience are shown in Table 4.

Table 4: Descriptive Statistic of Mathematic Resilience

Item	Mean	Variance	Std. Deviation	CI 95%		Skewness	Kurtosis
				Upper	Lower		
V1	3.373	0.492	0.701	3.441	3.304	0.993	-1.011
V2	2.321	0.872	0.934	2.412	2.230	-0.843	0.181
V3	2.731	0.885	0.941	2.823	2.639	-0.805	-0.282
V4	1.842	0.782	0.884	1.928	1.756	0.101	0.897
S5	1.968	0.893	0.945	2.060	1.876	-0.392	0.719
S6	3.351	0.585	0.765	3.425	3.276	1.132	-1.157
S7	2.094	0.823	0.907	2.182	2.005	-0.316	0.614
G8	3.133	0.814	0.902	3.221	3.045	0.064	-0.896
G9	2.099	1.079	1.039	2.200	1.997	-0.869	0.560
G10	3.247	0.577	0.760	3.321	3.173	0.880	-0.957
G11	1.973	0.868	0.932	2.064	1.882	-0.248	0.755
P12	3.343	0.790	0.889	3.430	3.256	1.022	-1.346
P13	1.938	0.910	0.954	2.031	1.845	-0.216	0.829
P14	3.560	0.524	0.724	3.631	3.490	3.428	-1.864
P15	1.788	0.767	0.876	1.873	1.702	0.555	1.072

The analysis results show variation in the distribution characteristics of the data across the study variables. Some variables tend to receive higher responses, with mean values approaching the maximum score, while others show lower mean values. There is evidence of non-normal distribution in several variables, either skewed to the left or right, with varying degrees of kurtosis from a normal distribution. The variability in the data across variables is also quite diverse, indicating differences in the spread of responses. The use of the entire response scale is evident from the range of values that span from the minimum to the maximum.

Findings Related to EFA of the Scale

EFA was conducted to examine the factor structure of the developed mathematical resilience instrument. This analysis aimed to empirically identify emerging factors from the data and assess their alignment with the theoretically designed construct. The process served as a foundational step in understanding inter-item relationship patterns before proceeding with model confirmation.

The Kaiser-Meyer-Olkin (KMO) test results in Table 5 above indicate that the data are highly suitable for factor analysis. The overall KMO value, which is high, suggests that the correlations among the variables are sufficiently strong and adequate for forming meaningful factors. The overall KMO measure of 0.854 demonstrates excellent sample adequacy, well above the recommended threshold of 0.50, confirming that the data is highly appropriate for factor analysis. Most individual items show strong sample adequacy, with values exceeding 0.70, indicating strong intercorrelations among the variables and supporting the factorability of the data set. These results provide confidence in proceeding with exploratory factor analysis to identify the underlying structure. The strong overall KMO value further reaffirms the appropriateness of the data set for identifying meaningful latent constructs.

Table 5: KMO Test Result

Indicator	MSA
V1	**0.857**
V2	**0.814**
V3	**0.522**
V4	**0.892**
S5	**0.864**
S6	**0.834**
S7	**0.879**
G8	**0.729**
G9	**0.867**
G10	**0.805**
G11	**0.874**
P12	**0.869**
P13	**0.880**
P14	**0.849**
P15	**0.876**
Overall	**0.854**

Table 6: Bartlett's Test Result

X^2	df	p
938.182	**105**	**< .001**

The results of Bartlett's Test presented in Table 6 show a X^2 value of 938.182 with 105 degrees of freedom (df) and a very small p value of < .001. Bartlett's Test is used to test the hypothesis that the correlation matrix is an identity matrix, which would indicate that there are no linear relationships between the variables in the data. The very small p-value, well below the commonly accepted significance level of 0.05, indicates that the null hypothesis (that the correlation matrix is an identity matrix) can be rejected. In other words, the results demonstrate that there are significant correlations between the variables in the data, supporting the feasibility of proceeding with factor analysis.

Furthermore, the high X^2 value and the very small p value indicate that the data possesses a strong structure, and factor analysis can be conducted with high validity. This result from Bartlett's Test supports the decision to proceed to the next step in factor analysis, namely identifying the underlying factors of the data. Thus, the test results further strengthen the confidence that this data is suitable for further analysis using exploratory factor analysis to uncover latent constructs. The strong result from Bartlett's Test specifically suggests that there are meaningful patterns of relationships between the variables that can be effectively modelled through factor analysis.

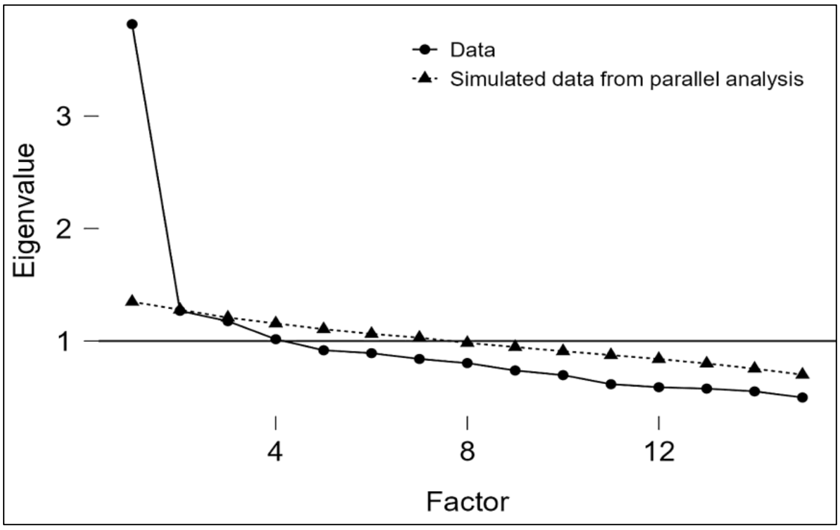

Figure 2: Screen Plot

Parallel analysis of the scree plot, comparing actual data eigenvalues (solid line) with simulated random data (dashed line). As shown in Figure 2, the eigenvalue curve reveals a clear inflection after the third factor, supporting the three-factor retention indicated by parallel analysis. The visual separation between actual and simulated eigenvalues further confirms that only the first three components explain meaningful variance. The first three factors show actual eigenvalues above the simulated line, indicating meaningful variance explanation. From Factor 4 onward, actual eigenvalues fall below the simulated line, suggesting insignificant contributions. The steep post-third factor decline further confirms three factors as optimal. This three-factor solution captures substantive dimensions while avoiding noise extraction, demonstrating robust factor structure.

Findings Related to CFA of the Scale

Following the factor structure exploration through EFA, the analysis advanced to CFA to evaluate the degree of fit between the theoretical model and the empirical data. The CFA focused specifically on testing construct validity and examining the strength of relationships among indicators within each factor, while statistically verifying the acceptability of the proposed model. In particular, indicators with the lowest values contribute minimally to the measurement of the construct, which may negatively impact the overall validity and reliability of the model. To assess potential method effects, we examined the presence of correlated uniqueness and method-factor patterns by evaluating modification indices (MIs) related to error covariances among reverse-coded items. The MIs did not indicate substantial method variance, and no correlated uniqueness terms exceeded acceptable thresholds. As the four-factor first-order model demonstrated adequate fit without adding method factors, a separate method-factor model was not estimated. This supports that method effects were minimal and did not bias the measurement structure.

The CFA employed a first-order factor model in which each of the four latent constructs (Value, Struggle, Growth, and Perseverance) was represented by its respective observed indicators. Maximum Likelihood (ML) estimation was used, as it is appropriate for continuous Likert-scale data and provides robust parameter estimation with samples exceeding 200. Model evaluation included multiple goodness-of-fit indices (CFI, TLI, NFI, RMSEA, SRMR, and χ^2/df) to ensure a comprehensive assessment of model adequacy. Additionally, modification indices (MIs) were examined to detect theoretically justifiable improvements in model fit; however, no cross-loadings or error covariances were added because the initial model met acceptable thresholds without substantive modifications.

Based on the loading factor calculations in Table 7 above, Composite Reliability (CR), and Fornell-Larcker ($\sqrt{\text{AVE}}$), the factors of Value, Struggle, and Growth demonstrate favorable results. All indicators in these three factors have loading factors greater than 0.70, indicating a strong contribution to their respective factors. Additionally, the CR values greater than 0.70, and $\sqrt{\text{AVE}}$ values for all three factors suggest that these factors exhibit adequate reliability and convergent validity, confirming that they are both valid and reliable (55–57). Figure 3 presents the standardized factor loadings for each indicator, offering a graphical overview of the measurement model's structure. The figure visually confirms that items loading on Value, Struggle, and Growth demonstrate strong loadings, whereas the Perseverance factor shows comparatively lower contributions, consistent with the statistical results.

Table 7: Validity and Reliability

	Loading Factors			CR	√AVE
Factor	**Indicator**	**Estimate**	**p**		
Value	V1	0.80	< .001	0.92	0.86
	V2	0.88	< .001		
	V3	0.99	0.235		
	V4	0.76	< .001		
Struggle	S5	0.64	< .001	0.79	0.75
	S6	0.85	< .001		
	S7	0.74	< .001		
Growth	G8	0.94	< .001	0.88	0.80
	G9	0.85	< .001		
	G10	0.78	< .001		
	G11	0.60	< .001		
Perseverance	P12	0.73	< .001	0.78	0.69
	P13	0.59	< .001		
	P14	0.74	< .001		
	P15	0.68	< .001		

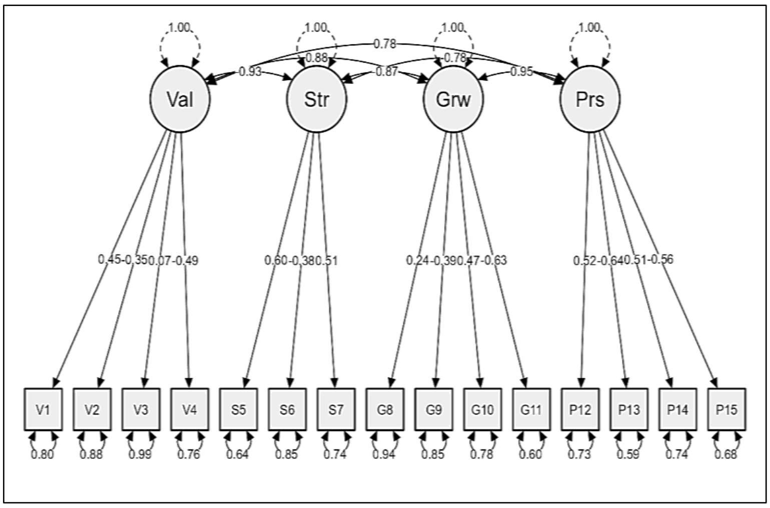

Figure 3: Factors Loadings in CFA

Therefore, no modifications or reductions in indicators are necessary for these factors. However, for the Perseverance factor, although the CR (0.78) meets the criterion, the √AVE indicates suboptimal convergent validity, suggesting that the Perseverance factor is less valid compared to the others. The indicator P13 has a lower loading factor (0.590) compared to the other indicators, suggesting that its contribution to the Perseverance factor is less significant.

Table 8: Chi-square Test Result

Model	X^2	df	p
Baseline model	**954.283**	**105**	
Factor model	**158.617**	**84**	**< .001**

The chi-square test results, as shown in Table 8, indicate that the proposed factor model (χ^2 = 158.617, df = 84) provides a significant improvement over the baseline model (χ^2 = 954.283, df = 105), with a p value of < 0.001, indicating a statistically significant difference. Although a significant chi-square result traditionally suggests a lack of perfect fit, the χ^2/df ratio of 1.89 (below the threshold of 3) and support from other fit indices suggest that the model is generally acceptable. It is important to consider the chi-square test's sensitivity to large sample sizes; thus, model adequacy should not be judged based solely on this result but should instead take into account a comprehensive evaluation of various goodness-of-fit indices.

Table 9: Fit Indices Test Result

Index	Value
Comparative Fit Index (CFI)	**0.912**
Tucker-Lewis Index (TLI)	**0.890**
Bentler-Bonett Normed Fit Index (NFI)	**0.834**
Root mean square error of approximation (RMSEA)	**0.047**
Standardized root mean square residual (SRMR)	**0.045**

Based on the goodness-of-fit analysis in Table 9 above, the measurement model demonstrates an adequate level of fit to the empirical data. The CFI value of 0.912 and the TLI value of 0.890 exceed the minimum recommended threshold of 0.90, indicating a good model fit. Although the NFI value of 0.834 falls slightly below the ideal standard of 0.90, it remains acceptable within the context of social science research. Regarding badness-of-fit measures, the RMSEA of 0.047 (90% CI) and the SRMR of 0.045 are both well below the critical value of 0.08, indicating low residual error and good model-data fit. Overall, the combination of these results supports the validity of the proposed measurement model, although there remains room for improvement, particularly in enhancing the NFI value through potential model modifications or the addition of key indicators.

Table 10: R-Squared Test Result

Indicator	R^2
V1	**0.200**
V2	**0.120**
V3	**0.005**
V4	**0.241**
S5	**0.364**
S6	**0.148**
S7	**0.262**
G8	**0.059**
G9	**0.153**
G10	**0.217**
G11	**0.403**
P12	**0.275**
P13	**0.414**
P14	**0.261**
P15	**0.318**

The R-squared test results in Table 10 reveal variation in the strength of the relationships between indicators and their latent constructs. Several indicators exhibit relatively high values, indicating that a substantial portion of their variance is explained by the underlying construct, thus confirming that these indicators are functioning well in measuring the intended construct. However, some indicators show very low R-squared values, approaching zero. Such low values suggest that these indicators have little meaningful relationship with their respective latent constructs.

Discussion

This study successfully validates a four-factor mathematical resilience model within the Indonesian educational context. The validated model, comprising Value, Struggle, Growth, and Perseverance, provides a crucial framework for understanding students' approaches to mathematics. These findings support the development of mathematics-specific resilience measurement tools. Theoretically, this research reinforces the significance of non-cognitive factors in mathematics education (31, 58). The high reliability of the Value dimension (CR=0.92) confirms its relevance in mathematical pedagogy. Thus, this study establishes an empirical foundation for resilience-based learning approaches.

Mathematical reciprocity is also shaped by cultural and educational contexts. In collectivist cultures such as Indonesia, reciprocity typically emerges through collaborative engagement, shared reasoning, and peer support during problem-solving. In contrast, more individualistic contexts may express reciprocity through independent exchange of strategies or competitive comparison of solutions. These cultural variations highlight that reciprocal mathematical interaction is not universal but context-dependent. Therefore, the patterns observed in this study reflect the collaborative orientation of Indonesian classrooms, and future cross-cultural studies may reveal differing manifestations of reciprocity.

The MRS-ES serves as an effective diagnostic tool, enabling educators to identify specific areas for development. For instance, students with low Value scores benefit from real-world mathematics applications, while those with weak Growth require activities emphasizing learning from mistakes. Such targeted interventions prove more effective than generic resilience training. The Struggle dimension's reliability (CR=0.79) supports contemporary 'productive struggle' pedagogical approaches (59). This tool empowers teachers to deliver precisely tailored interventions, enhancing instructional effectiveness.

The strong results for the Growth dimension (CR=0.88) reinforce the application of growth mindset theory in mathematics (58). These findings validate process-focused feedback practices in mathematics instruction. However, the lower validity of the Perseverance dimension indicates challenges in measuring mathematics-specific persistence. Students may struggle to differentiate general perseverance from mathematical persistence, suggesting the need for measurement refinement. Instrument adaptation is necessary to better capture the unique characteristics of mathematical perseverance. These findings highlight the importance of domain-specific resilience assessment tools.

Validated scale items can inform the creation of targeted learning activities. A holistic approach integrating cognitive and affective aspects would better support comprehensive student development. The observed variation in resilience profiles among students underscores the value of differentiated instruction. Teaching strategies should be adapted to address individual resilience needs. Incorporating resilience assessment into learning evaluations would provide a more complete understanding of student progress. Consequently, mathematics education practices could become more effective in fostering student resilience.

The cross-sectional design limits causal inferences about resilience development over time. The research cannot determine whether enhanced resilience leads to improved achievement or vice versa. Potential self-report bias, particularly for Perseverance items, may affect result validity. Incorporating multiple assessment methods, such as teacher observations, could strengthen findings. Cultural adaptation is also necessary to fully understand mathematical resilience in the Indonesian context. These limitations indicate the need for more comprehensive follow-up studies.

Instrument validation should be expanded across diverse Indonesian regions to test generalizability. Integrating MRS-ES data with academic performance metrics could clarify resilience-achievement relationships. Intervention studies are needed to assess the effectiveness of resilience-building programs. Qualitative research could further explore cultural conceptualizations of mathematical resilience. These steps would enhance the practical utility of resilience measurement tools. Future research could thereby make more substantial contributions to mathematics education.

Mathematical resilience encourages persistence through challenges rather than avoidance (60). Previous research demonstrates its relationship with learning motivation and academic achievement (31). Growth mindset plays a pivotal role in developing this resilience (58). Students' conceptions of mathematical ability significantly influence their resilience (59). Resilience constitutes not merely a supplement, but rather an essential component of mathematics learning. Its development should be prioritized in mathematics education. This study provides a validated framework for assessing mathematical resilience in Indonesia. Incorporating resilience assessment into teaching practices can enhance instructional effectiveness. Further research is necessary to address the current study limitations. Intervention programs based on these findings would benefit educational practice. Developing mathematical resilience is crucial for long-term student engagement and success. These findings advocate for transforming mathematics education to better address psychological aspects of learning.

Conclusion

In conclusion, while this study makes significant strides in measuring mathematical resilience in Indonesia, it also opens new avenues for research and practice. The MRS-ES provides a scientifically validated tool that can inform both classroom instruction and education policy, but its full potential will only be realized through continued refinement and application. By addressing the current limitations and building on the study's strengths, future research can further illuminate how to best support Indonesian students in developing the resilience needed for mathematical success. The findings ultimately underscore that while mathematical resilience has universal components, its measurement and cultivation must account for educational contexts. Future studies in this area would deepen our understanding of how non-cognitive factors influence mathematics learning and provide evidence-based strategies for fostering mathematical resilience across diverse educational settings.

Abbreviations

√AVE: Square Root of Average Variance Extracted, AVE: Average Variance Extracted, CFA: Confirmatory Factor Analysis, CFI: Comparative Fit Index, CR: Composite Reliability, CTT: Classical Test Theory, JASP: Jeffrey's Amazing Statistics Program, KMO: Kaiser-Meyer-Olkin Measure of Sampling Adequacy, NFI: Normed Fit Index.

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Author Contributions

Wafiq Nurul Huda: conceptualization, research design, data collection, drafting of the manuscript, E Kus Eddy Sartono: supervision, critical feedback, contributed to the refinement of the theoretical framework and methodology, Bambang Saptono: offered expertise in the validation of the research instruments, the interpretation of findings, Farah Rizkita Putri: statistical analyses, including conducting the exploratory factor analysis (EFA), confirmatory factor analysis (CFA) using JASP, assisted in the final editing, formatting of the manuscript, Zhuldiz Anay: proofreading, language refinement, enhancement of the manuscript's readability. All authors have read and approved the final version of the manuscript.

Conflict of Interest

The authors declare no conflicts of interest.

Declaration of Artificial Intelligence (AI) Assistance

The authors acknowledge the use of DeepL Translator and Grammarly solely for the purpose of improving language readability and grammar. All scientific content, data analysis, intellectual reasoning, and conclusions remain the sole contribution of the human authors.

Ethics Approval

This paper is not currently being considered for publication elsewhere. This study followed ethical guidelines, obtained consent, and ensured voluntary participation. This study obtained permission from the relevant institute to carry out this work in an honest and ethical manner.

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