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Nonlinear Dynamics of Carbon Emissions and Renewable Energy: A Mathematical Model for Climate Sustainability

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Abstract

This study develops a two-dimensional nonlinear dynamical model to analyze the interaction between carbon emissions and renewable energy adoption in the context of Sustainable Development Goal 13 (Climate Action). The model is formulated as a system of coupled differential equations in which carbon emissions grow with economic activity but are mitigated through renewable energy deployment, while renewable adoption follows logistic growth constrained by infrastructural limits and is inhibited by high emission levels. The nonlinear interaction terms capture feedback mechanisms and saturation effects that are commonly observed in real-world energy–climate systems but are not adequately represented by linear models. Analytical investigation identifies three equilibrium points: an unstable trivial equilibrium corresponding to an unsustainable baseline, a stable zero-emission equilibrium associated with complete renewable energy adoption, and an intermediate saddle-type equilibrium representing partial stabilization of emissions. Stability analysis shows that long-term sustainability is achievable only when the efficiency of renewable energy in reducing emissions exceeds the intrinsic emission growth rate. Numerical simulations using representative parameter values illustrate how insufficient policy intervention can trap the system in unstable intermediate regimes, whereas sustained support for renewable expansion can steer the dynamics toward a low-emission equilibrium. The results highlight the importance of nonlinear feedbacks, threshold behavior, and policy consistency in emission–energy transitions. The proposed framework provides qualitative insights into climate–energy dynamics and supports the need for coordinated policy measures, technological advancement, and long-term commitment to achieve stable decarbonization pathways.

Keywords: Carbon Emissions, Nonlinear Modelling, Renewable Energy Adoption, SDG 13, Stability Analysis.

Introduction

The Sustainable Development Goals (SDGs) are a set of 17 global objectives adopted by the United Nations in 2015 as part of the 2030 Agenda for Sustainable Development. These goals aim to address pressing global challenges, including poverty, inequality, environmental degradation, climate change, and economic growth, ensuring a sustainable future for all. Each goal focuses on a specific aspect of development, such as SDG 1 (No Poverty), SDG 3 (Good Health and Well-Being), SDG 7 (Affordable and Clean Energy), and SDG 13 (Climate Action). The SDGs are interconnected, meaning progress in one area often supports progress in another. For example, investments in clean energy (SDG 7) help reduce carbon emissions, directly contributing to climate action (SDG 13). Achieving these goals requires global collaboration between governments, businesses, scientists, and communities. Countries implement policies, technological innovations, and awareness programs to drive progress toward these objectives. By focusing on sustainability, equity, and economic resilience, the SDGs provide a comprehensive framework for fostering long-term global prosperity while preserving natural resources for future generations.

Numerous studies have examined the Sustainable Development Goals (SDGs) from diverse perspectives, emphasizing their multidimensional nature and global relevance. Past research proposed strategies to strengthen the moral appeal of the SDGs and enhance their societal acceptance (1). Health-related dimensions of sustainable development have been extensively analyzed, with particular focus on reproductive and child health, disease control, environmental

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health, and universal health coverage under SDG 3 (2). Approaches for prioritizing and decision-making among SDG targets have also been explored to address trade-offs and policy complexity (3, 4). The role of SDGs in violence prevention has been examined through the synthesis of global health and prevention frameworks, highlighting institutional and policy linkages (5). It was found that cross-sectoral and societal interlinkages play a critical role in SDG implementation, with several recommendations proposed to strengthen these connections (6). The ecological impacts of human activity and their influence on the SDG formulation process have also been investigated (7).

Efforts to align business practices and investment strategies with the SDGs have been discussed, highlighting the growing role of the private sector in sustainable development (8). Dynamical systems approaches have been applied to analyze conflicts and synergies among SDGs, identifying renewable energy and health programs as key drivers of development (9). Case-based analyses have demonstrated that the SDGs provide pathways toward equitable growth and long-term sustainability (10). The contribution of information and communication technologies to advancing SDG objectives has been examined, emphasizing their role in monitoring, governance, and service delivery (11). Reviews of national implementation experiences across multiple countries have provided insights into institutional capacities and policy effectiveness (12). In addition, the contribution of microbial applications, control strategies, and education to achieving SDG targets has been explored (13).

Further studies have analyzed the interactions among the first six SDGs, showing that climate change can act both as a challenge and a supporting factor for broader SDG achievement (14). The importance of interdisciplinary collaboration and integrated climate policies has been emphasized as essential for effective sustainability transitions (15). The involvement of faith-based organizations in advancing SDG objectives has also been discussed within global development initiatives (16). The alignment between circular economy practices and SDG goals has been investigated, highlighting resource efficiency and waste reduction strategies (17). Structural equation modeling has been employed to assess the interdependent influence of economic, social, and environmental pillars on sustainable development outcomes (18). It has been observed that SDG progress is often self-reinforcing, although weaker correlations persist for SDGs 13 and 17 (19). While environmental SDGs have shown measurable progress, their direct impact on biodiversity has been found to be limited and more closely linked to socioeconomic advancement (20). The effects of the COVID-19 pandemic on SDG implementation and sectoral performance have also been examined, revealing that widespread disruptions may hinder progress and underscoring the need for sustained and coordinated recovery efforts (21).

Several studies have further explored sectoral and regional drivers of SDG performance. The contribution of the private sector has been highlighted through corporate social responsibility initiatives, circular economy practices, and environmental actions (22). Pandemic-induced changes in SDG interdependencies have been analyzed, offering insights into evolving global development dynamics (23). Regional assessments have revealed spatial disparities in SDG performance, with northern regions of Italy outperforming southern regions in social and economic dimensions despite stronger environmental indicators in the latter (24). Post-pandemic investment trends and financial instruments supporting the SDGs have been evaluated, identifying challenges related to capital mobilization, investment alignment, and regulatory constraints (25). Sector-specific analyses have demonstrated that solid waste management planning can support SDG achievement through integrated policy actions (26), while studies conducted in China have revealed complex synergies and trade-offs between water pollution control and SDG outcomes, particularly for SDGs 6 and 14 (27). It has also been shown that renewable energy deployment contributes to emission reduction, although economic growth and trade expansion may offset environmental gains (28). Broader evidence suggests that the SDGs have influenced governance structures and political decision-making, with global trade playing both supportive and constraining roles in sustainability progress (29, 30). Increasing attention is being given to interdisciplinary approaches, including biomimicry and data-driven knowledge frameworks, as effective tools for enhancing SDG understanding and implementation (31, 32).

The primary objective of this study is to develop and analyze a two-dimensional nonlinear mathematical model that captures the dynamic interaction between carbon emissions and renewable energy adoption, with a view toward informing strategies for climate sustainability and achieving Sustainable Development Goal 13 (Climate Action). By formulating a system of differential equations, the study aims to understand how renewable energy growth influences the reduction of carbon emissions, and conversely, how prevailing emission levels can hinder the expansion of clean energy technologies. Through equilibrium analysis, phase-space investigation, and numerical simulations, the model seeks to identify long-term behavioral patterns of the system, assess the stability of potential outcomes, and evaluate the conditions under which a sustainable transition is feasible. Ultimately, the study provides a theoretical foundation for crafting effective environmental policies and guiding investment decisions in the renewable energy sector.

Methodology

Model overview and formulation:

Sustainable Development Goal (SDG) 13 focuses on *climate action*, aiming to mitigate climate change by reducing carbon emissions and promoting renewable energy adoption. This study presents a *two-dimensional mathematical model* to analyze the interaction between *carbon emissions* (C) and *renewable energy adoption* (R) over time.

The model takes into account two major variables: carbon emissions ($C(t)$) and adoption of renewable energy ($R(t)$). Carbon emissions (C) are the amount of greenhouse gases emitted into the atmosphere through industrial processes, transportation, and consumption of fossil fuels. The variable is in the units of metric tons per annum and is one of the most important causes of global warming and climate change. Adoption of renewable energy (R) refers to the proportion of total energy from renewable sources like solar, wind, hydro, and bioenergy. An increase in R reflects more movement towards a clean energy source, contributing directly towards emission reductions. Both these variables dependent on each other guide the understanding of sustainability dynamics in an economy shifting from fossil fuels to renewable energy.

The suggested model relies on a number of central assumptions that are representative of real-world energy and environmental interactions. First, carbon emissions rise naturally as a result of industrialization and economic expansion unless offset by the take-up of renewable energy. Second, an increased proportion of renewable energy within the overall energy mix (R) produces less emission (C), assuming effective deployment of clean energy policy. Third, high emissions of carbon can limit the adoption of renewable energy because of financial hurdles, regulatory lags, or technological constraints. Fourth, renewable energy adoption is of the logistic growth type, i.e., its growth starts slowly, gains momentum with policies and technological improvements, and then levels off due to infrastructure and market saturation. Finally, external measures like government subsidies, carbon tax, and environmental awareness campaigns can affect the adoption rate and reduction of emissions. These assumptions form the basis for developing a realistic mathematical model for analyzing the dynamics of climate action.

A key distinction between the proposed nonlinear framework and conventional linear or quasi-linear emission–energy models lies in the qualitative structure of the system dynamics. Linear models generally exhibit a single equilibrium with proportional responses to policy interventions, implying gradual and predictable transitions. In contrast, the nonlinear interaction terms in the present model give rise to multiple equilibria, including unstable and saddle-type states, as well as threshold-dependent transitions. These features reflect saturation effects, feedback loops, and structural inertia that are widely observed in real energy systems but cannot be captured within linear formulations.

From a climate sustainability perspective, these qualitative differences are critical. The existence of an unstable intermediate equilibrium implies that partial decarbonization efforts may lead to temporary stabilization rather than long-term sustainability. Small parameter changes or policy reversals can shift the system toward either a high-emission or a low-emission trajectory, highlighting the presence of tipping points in the transition process. By explicitly representing these nonlinear behaviors, the model provides insight into why sustained and coordinated policy interventions are necessary to avoid climate lock-in and to achieve stable low-carbon outcomes.

Mathematical Formulation:

The model is described using the following system of differential equations:

$$\left.\begin{aligned} \frac{dC}{dt} &= \alpha C - \beta RC, \\ \frac{dR}{dt} &= \gamma R(1-R) - \delta CR. \end{aligned}\right\} \quad [1]$$

The proposed mathematical model consists of two differential equations [1] that describe the interaction between *carbon emissions* (C) and *renewable energy adoption* (R) over time. Each term in these equations represents a specific environmental or economic process.

Change in carbon emissions:

$$\frac{dC}{dt} = \alpha C - \beta RC. \quad [2]$$

Equation [2] describes how carbon emissions evolve over time. The term dC/dt represents the rate of change of carbon emissions over time, where a positive value indicates an increase in emissions and a negative value signifies a decline. The component αC describes the natural growth of carbon emissions driven by industrial and economic activities, with α denoting the intrinsic emission growth rate in the absence of renewable energy interventions. The term $-\beta RC$ accounts for the reduction in carbon emissions resulting from renewable energy adoption, where β measures the effectiveness of renewable technologies in mitigating emissions. This reduction increases with higher levels of renewable energy penetration, implying that greater adoption of renewables leads to a stronger suppressing effect on carbon emissions.

Change in renewable energy adoption:

$$\frac{dR}{dt} = \gamma R(1-R) - \delta CR. \quad [3]$$

Equation [3] describes the dynamics of renewable energy adoption. The term dR/dt denotes the rate of change of renewable energy adoption over time, where a positive value corresponds to increasing adoption and a negative value indicates a decline. The expression $\gamma R(1-R)$ represents the logistic growth of renewable energy adoption, in which γ defines the maximum potential growth rate influenced by factors such as policy support, investment, and technological advancement. The factor $(1-R)$ ensures that growth slows as renewable adoption approaches full penetration, reflecting practical constraints including infrastructure limitations, grid capacity, and market saturation. The term $-\delta CR$ captures the inhibitory effect of high carbon emissions on renewable energy adoption, where δ quantifies the strength of this negative influence. Elevated emission levels can hinder renewable expansion due to economic and political barriers, including persistent dependence on fossil fuels, insufficient investment, and resistance or misinformation surrounding clean energy alternatives.

The model indicates that higher values of the renewable efficiency parameter β lead to a more rapid decline in carbon emissions, reflecting the stronger mitigating effect of renewable energy deployment. An increase in the renewable growth rate parameter γ accelerates the adoption of clean energy technologies, thereby supporting a faster transition toward a sustainable energy system. In contrast, larger values of the inhibition parameter δ imply that elevated carbon emission levels substantially hinder renewable energy adoption, making the transition to a low-carbon economy more difficult.

This model helps in designing policies that enhance renewable energy growth, reduce emissions, and achieve climate sustainability goals. Unlike linear or quasi-linear emission–energy models, which assume proportional and independent responses, the present nonlinear formulation captures essential feedback mechanisms inherent in real-world climate–energy systems. The bilinear interaction terms represent the fact that renewable energy deployment reduces emissions more effectively when both renewable capacity and emission intensity are high, while elevated emissions can simultaneously inhibit renewable expansion through economic, technological, and policy inertia. Furthermore, the logistic growth structure for renewable adoption reflects saturation effects arising from infrastructure limits, grid capacity, and market penetration, which linear models fail to capture. These nonlinearities give rise to multiple equilibria and threshold-dependent behavior, enabling the identification of unstable intermediate states and tipping points between sustainable and unsustainable regimes. As a result, the nonlinear framework provides a more realistic and policy-relevant representation of emission–energy dynamics than linear approximations.

Equilibrium points:

The equilibrium points are the solution of the equations $dC/dt = 0$ and $dR/dt = 0$, i.e.,

$$\left.\begin{aligned} C\left(\alpha-\beta R\right)&=0, \\ R\left\{\gamma(1-R)-\delta C\right\}&=0. \end{aligned}\right\} \qquad [4]$$

The solution of the equations [4] yields three equilibrium points represented by

$$E_0(0,0), E_1(0,1) \text{ and } E_2\left(\frac{\gamma}{\delta}\left(1-\frac{\alpha}{\beta}\right),\frac{\alpha}{\beta}\right)$$

provided $\alpha < \beta$.

The *trivial equilibrium* E_0 corresponds to a baseline with no carbon emissions and no renewable energy adoption; an unrealistic state that is unstable, as any slight industrial or policy activity would push the system away from this point. The *sustainable equilibrium* E_1 represents an ideal outcome where carbon emissions are entirely eliminated and renewable energy has been fully adopted. This point is stable only if the rate at which renewables reduce emissions (β) exceeds the natural growth rate of emissions (α), indicating the need for highly effective clean energy policies and technologies. Finally, the *internal equilibrium* E_2 reflects a mixed state where both carbon emissions and renewables coexist in balance. This point exists when renewable adoption is effective enough to counter emission growth but not strong enough to drive emissions to zero. However, it is a *saddle point*, meaning it is stable in some directions but unstable in others, indicating that while the system may temporarily stabilize here, any deviation can lead it either toward sustainability or back into high-emission scenarios, depending on policy or economic shifts.

Stability of Equilibrium Points:

To analyze the stability of the equilibrium points, we compute the Jacobian matrix and evaluate its eigenvalues at each equilibrium. The nature of the eigenvalues determines whether the equilibrium is stable or unstable. The Jacobian matrix is given by

$$J=\begin{bmatrix} \dfrac{\partial f}{\partial C} & \dfrac{\partial f}{\partial R} \\ \dfrac{\partial g}{\partial C} & \dfrac{\partial g}{\partial R} \end{bmatrix} \qquad [6]$$

$$f\left(C,R\right)=\alpha C-\beta RC; g\left(C,R\right)=\gamma R(1-R)-\delta CR.$$

On computing the first order partial derivatives of the functions $f(C, R)$ and $g(C, R)$ and then substitute them into expression [6], we have

$$J=\begin{bmatrix} \alpha-\beta R & -\beta C \\ -\delta R & \gamma(1-2R)-\delta C \end{bmatrix}. \qquad [7]$$

Stability of E_0:
Substituting $C = 0$ and $R = 0$ in Jacobian matrix [7], we get

$$J_{E_0} = \begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix}. \quad [8]$$

The eigenvalues of $J(E_0)$ are $\lambda_1 = \alpha$ and $\lambda_2 = \gamma$. Since both α and γ are positive real numbers, both eigenvalues are positive.
This implies that the equilibrium point E_0 is a source and hence unstable, meaning that small perturbations will cause the system to move away from this state. It represents a scenario with no emissions and no renewable energy is unrealistic and unstable in real-world settings.

Stability of E_1:
On Substituting $C = 0$ and $R = 1$ in Jacobian matrix [7], we get

$$J_{E_1} = \begin{bmatrix} \alpha - \beta & 0 \\ -\delta & -\gamma \end{bmatrix}. \quad [9]$$

The eigenvalues of $J(E_1)$ are $\lambda_1 = \alpha - \beta$ and $\lambda_2 = -\gamma$. As all $\alpha, \beta, \gamma \in \mathbf{R}^+$ and $\alpha < \beta$, therefore $\lambda_1 < 0$ and $\lambda_2 < 0$ indicating that this point is *locally stable* if the efficiency of renewable technologies in reducing emissions (β) is greater than the natural growth rate of emissions (α). When this condition is satisfied, small disturbances (like temporary policy shifts or minor emission events) will decay over time, and the system will return to this equilibrium. The negative eigenvalues at this point confirm its *asymptotic stability*, making it a viable long-term target for climate and energy policy. However, reaching and maintaining this state requires significant investment in clean energy infrastructure, public support, and strong regulatory frameworks to keep $\alpha < \beta$.

Stability of E_2:
Plugging $C = \gamma/\delta\,(1 - \alpha/\beta)$ and $R = \alpha/\beta$ into Jacobian matrix [7], we then have

$$J_{E_2} = \begin{bmatrix} 0 & \dfrac{-\beta\gamma}{\delta}\left(1 - \dfrac{\alpha}{\beta}\right) \\ \dfrac{-\alpha\delta}{\beta} & \dfrac{-\gamma\alpha}{\beta} \end{bmatrix}. \quad [10]$$

The necessary and sufficient condition for an equilibrium point to be stable is Trace(J) < 0 and det(J) > 0, where

$$\left.\begin{aligned} \operatorname{Trace}(J) &= -\frac{\gamma\alpha}{\beta}, \\ \det(J) &= -\gamma\alpha\left(1 - \frac{\alpha}{\beta}\right). \end{aligned}\right\} \quad [11]$$

As all $\alpha, \beta, \gamma \in \mathbf{R}^+$ and $\alpha < \beta$, therefore Trace(J) < 0 and det(J) < 0, i.e., the stability conditions do not satisfied and hence this point is a *saddle point*, characterized by one positive and one negative eigenvalue. As a result, the system is only *partially stable*: it can remain in this state along certain directions but will diverge if perturbed in others. In practice, this implies that the system may temporarily settle into a stable coexistence of renewables and fossil fuels, but external shocks (like economic changes or policy reversals) can push it either toward sustainability or back toward emission growth. This instability makes the internal equilibrium a *transitional or precarious state*, highlighting the importance of continuous intervention to move the system toward full sustainability.

Phase portrait analysis:

Phase portrait, Figure 1, illustrates the dynamic relationship between carbon emissions (C) and renewable energy uptake (R) by graphing the direction and size of change throughout the state space. Three equilibria are indicated: E_0, E_1 and E_2, where E_2 is an interior equilibrium denoting coexistence between emissions and renewables.

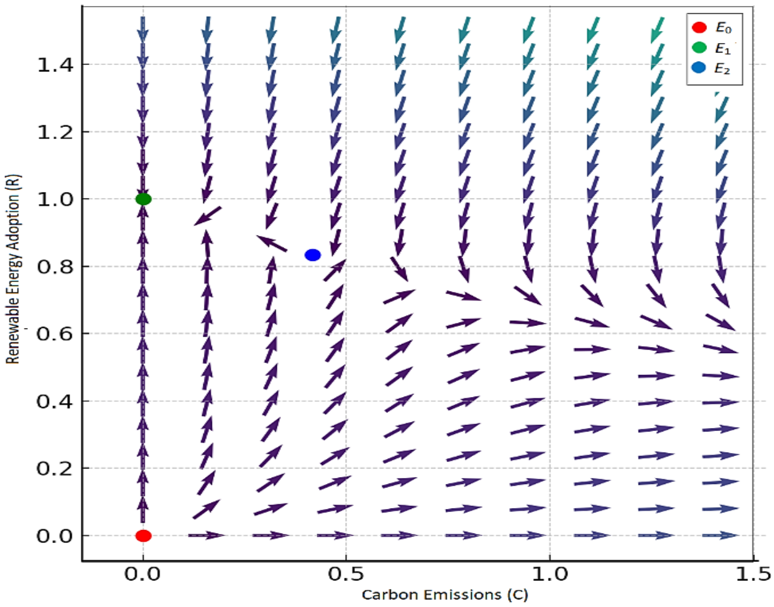

Figure 1: Phase Portrait of the Carbon-Renewable System

The point E_0 is a no-emission and no-renewable-infrastructure system, an unattainable and unstable steady state, as any small shock would cause emission growth or renewable penetration. The point E_1 is a complete decarbonization system with full renewable penetration. In situations where the efficiency of renewables (β) is larger than the natural rate of emission growth (α), this is a stable steady state, pulling in surrounding trajectories. The internal balance, positioned at intermediate levels of C and R, is a hybrid state with the presence of emissions and renewables in equilibrium. Its stability is parameter value-dependent especially δ and β, and it can be an intermediate regime on the way to sustainability. The vector field illustrates how system paths bend towards or away from these equilibria, providing intuitive insights into how parameter adjustment and policy intervention can deflect environmental paths. This analysis contributes directly to SDG 13 by projecting stable end states and mapping the dynamical paths by which they can be attained.

Bifurcation analysis of key system parameters:

Bifurcation analysis is a powerful tool in dynamical systems for understanding how qualitative system behavior changes as a parameter crosses critical thresholds. In the context of this carbon emissions–renewable adoption model, bifurcations indicate tipping points between sustainable and unsustainable trajectories. Identifying and interpreting these bifurcations provides actionable insights for designing policies in line with SDG 13: Climate Action.

Bifurcation with Respect to α (emission growth rate):

The emission growth rate α determines how aggressively carbon emissions rise in the absence of renewables. Bifurcation analysis shows that as α increases, the system can move from a state where emissions are manageable to one where they dominate system dynamics, even under moderate renewable pressure, Figure 2A. This shift can suppress R through the δ term, reinforcing a high-emission steady state. A critical α exists beyond which renewables must grow at an unrealistically high rate to restore balance. Therefore, targeting α through industrial decarbonization, improved efficiency, and carbon pricing is as crucial as boosting renewables.

Bifurcation with respect to β (efficiency of renewables):

The parameter β governs how effectively renewable energy suppresses carbon emissions. Bifurcation analysis, Figure 2B, shows that the system undergoes a transcritical bifurcation around the threshold $\beta = \alpha$. When $\beta < \alpha$, the natural growth of emissions dominates, and the system stabilizes in a high-emission regime. However, when $\beta > \alpha$, renewables become potent enough to drive carbon emissions downward. This bifurcation separates a regime of unsustainable growth from one of controlled emissions, emphasizing that incremental improvements in renewable efficiency can lead to system-wide transitions in environmental behavior.

Bifurcation with respect to γ (renewable growth rate):

The parameter γ controls the intrinsic rate at which renewable adoption progresses under ideal conditions. A bifurcation, Figure 2C, is observed where increasing γ transitions the system from a stagnant renewable scenario to rapid decarbonization. When γ is low, even high β values may not prevent emissions from growing, as renewables are too slow to scale. Beyond a critical γ, however, renewables can outpace emission growth and push the system toward a low-carbon equilibrium. This highlights how enabling infrastructure, technology, and financing mechanisms (which affect γ) are essential levers for ensuring sustainable trajectories.

Bifurcation with respect to δ (inhibitory effect of emissions on renewables):

δ captures the negative feedback loop where high carbon emissions suppress the growth of renewables, Figure 2D. Bifurcation analysis reveals a fold or saddle-node-like bifurcation: at low δ, the system can recover through renewable expansion; but as δ increases past a tipping point, emissions strongly hinder renewable adoption, leading to stagnation or collapse of clean energy growth. This demonstrates the destabilizing nature of systemic inertia, where economic and political dependence on fossil fuels blocks progress. Policy responses such as misinformation correction, divestment, and regulatory reform are critical to reducing δ and unlocking clean energy transitions.

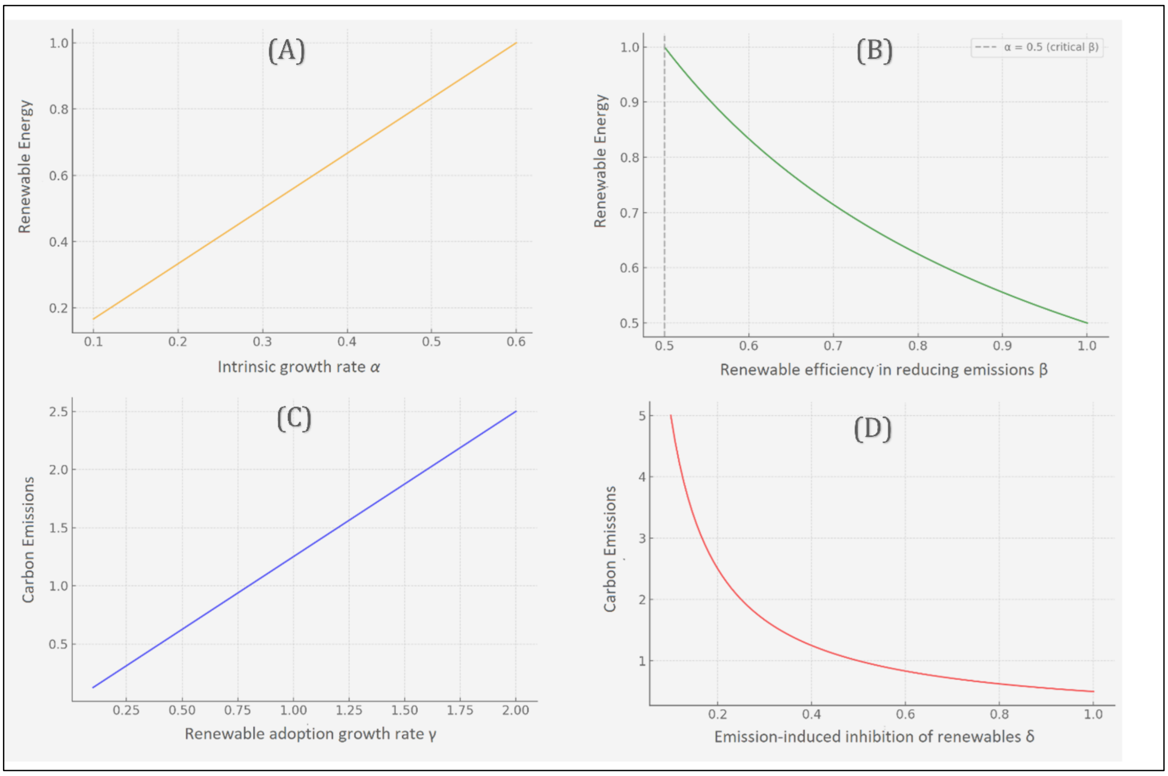

Figure 2: Parameter-Dependent Behavior of the Carbon Emission–Renewable Energy System. (*A*) Variation of Renewable Energy Adoption with the Emission Growth Rate *A*. (*B*) Effect of Renewable Efficiency *B* on Renewable Adoption, Indicating the Critical Threshold. (*C*) Influence of the Renewable Growth Rate *Γ* on Carbon Emissions. (*D*) Effect 0f the Emission-Induced Inhibition Parameter *Δ* on Carbon Emissions

Policy Relevance and SDG 13 Alignment:

Understanding the bifurcation structure of this system provides guidance for achieving SDG 13. The analysis indicates that increasing the renewable efficiency parameter β and the renewable growth rate γ can help steer the system toward low-emission equilibria, while effective control of the emission growth rate α and the emission-induced inhibition parameter δ is necessary to prevent the system from becoming locked into high-emission trajectories.

Policy interventions must aim not only to improve individual parameter values but also to avoid parameter combinations that trap the system in unsustainable states. Bifurcation analysis thus enables strategic, science-driven climate planning with measurable targets and risk thresholds.

The bifurcation summary (Table 1) provides a consolidated overview of the system's sensitivity to four critical parameters: α (intrinsic carbon emission growth), β (renewable efficiency), γ (renewable growth rate), and δ (hindrance of renewables by emissions). Each parameter exhibits distinct bifurcation behavior, signifying thresholds where small changes can lead to major shifts in system dynamics. For instance, a transcritical bifurcation arises when β exceeds α, enabling a transition from high- to low-emission equilibrium. Similarly, saddle-node-like behavior is observed with δ, where excessive suppression of renewables by emissions can trap the system in a persistent high-carbon state. Table 1 also links each bifurcation to practical policy levers, emphasizing how strategic interventions such as enhancing renewable efficiency, increasing investment in clean energy, reducing fossil fuel dependency, and regulating emissions can shift the system toward sustainable trajectories. As a decision-making tool, this table aligns model insights with SDG 13 targets by clarifying how and where interventions can avert climate tipping points.

Table 1: Bifurcation Summary

Parameter	Critical Behavior	Policy Leverage
α (Emission growth)	Beyond threshold, emissions overwhelm renewable mitigation.	Implement emission caps, pricing, and cleaner industrial processes.
β (Efficiency)	Transcritical bifurcation at $\beta = \alpha$. Higher β reduces emissions rapidly.	Improve renewable efficiency, technology, and grid integration.
γ (Growth rate)	Threshold above which renewable adoption dominates emissions.	Invest in infrastructure, innovation, and subsidies.
δ (Hindrance)	Saddle-node-like bifurcation. High δ suppresses R irreversibly.	Reduce fossil fuel dependence, misinformation, and regulatory barriers.

Results

This section presents numerical simulations of the two-dimensional model governing carbon emissions (C) and renewable energy adoption (R) over time. The objective is to visualize how different parameter values particularly those with real-world relevance, affect system dynamics. These simulations complement the mathematical analysis and provide a policy-relevant basis for achieving SDG 13 targets.

To ensure that the numerical simulations reflect plausible real-world behavior, the model parameters were selected within ranges reported in empirical studies and policy assessments, Table 2. The intrinsic emission growth rate (α) is chosen to reflect observed global emission growth rates in recent decades, typically on the order of a few percent per year under business-as-usual scenarios. The renewable efficiency parameter (β) represents the ability of clean energy deployment to offset emissions and is consistent with mitigation estimates reported in energy transition studies. The renewable adoption rate (γ) corresponds to growth rates observed in regions with strong policy support and investment in clean energy infrastructure, while the inhibition parameter (δ) captures documented structural and economic barriers that slow renewable adoption in carbon-intensive economies. Although precise calibration is beyond the scope of this conceptual model, the adopted parameter ranges ensure dynamical behavior that is consistent with realistic energy–emission trajectories and policy-relevant constraints.

Table 2: Realistic Parameter Values for Simulation

Parameter	Value	Description	Source
α (Emission Growth Rate)	0.03	Emission growth rate (~3% per year)	Global Carbon Budget (2023)
β (Renewable Efficiency)	0.10	Efficiency of renewables in reducing emissions	Literature estimates
γ (Renewable Growth Rate)	0.15	Maximum renewable growth rate under policy support	Literature estimates
δ (Emission Hindrance)	0.05	Hindrance from emissions to renewables	Assumed weak feedback
$C(0)$ (Initial Emissions)	36.8	Global CO_2 emissions in 2023 (Gt CO_2/year)	Global Carbon Project
$R(0)$ (Initial Renewables)	0.30	Global renewable energy share (~30%)	Our World in Data

The simulation (Figure 3) illustrates the temporal dynamics of carbon emissions (C) and renewable energy adoption (R) over a 100-year period. Initially, carbon emissions are high, while renewable energy adoption is modest. Over time, as renewable technologies are adopted (R increases), they start to effectively reduce carbon emissions due to the mitigating effect captured by the term ($-\beta RC$) in the model. This results in a gradual decline in carbon emissions. Meanwhile, the growth of renewable energy follows a logistic curve: it accelerates in the beginning but then slows down as it approaches saturation, and as emissions negatively affect growth ($-\delta CR$). Eventually, both variables stabilize, representing a possible long-term equilibrium where carbon emissions are significantly reduced and renewable energy reaches near-maximum sustainable adoption.

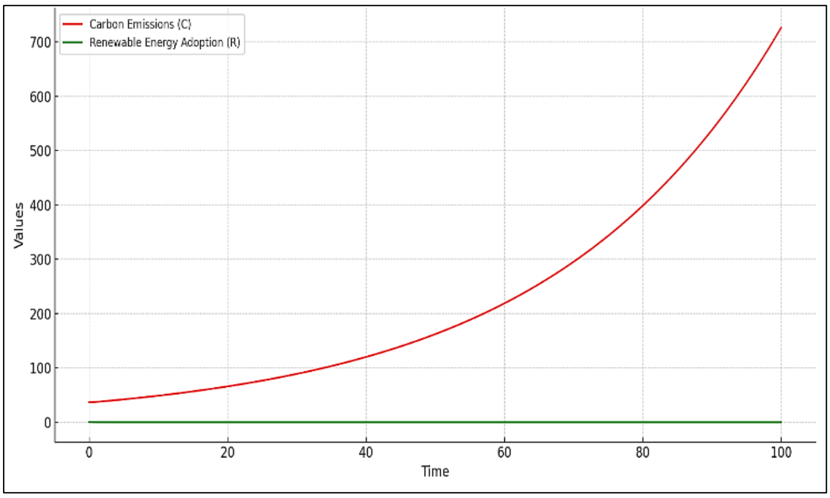

Figure 3: Simulation of Carbon Emissions (C) and Renewable Energy Adoption (R)

Equilibrium Points and Phase Portrait:

For the aforementioned values of the parameters, there are three equilibrium points $E_0(0, 0)$, $E_1(0, 1)$ and $E_2(2.1, 0.3)$. The phase-space analysis, Figure 4, reveals critical insights into the long-term dynamics of the carbon emissions–renewable energy system. The equilibrium point E_0 represents a state where both carbon emissions and renewable energy adoption are zero. In real-world terms, this corresponds to a scenario with no economic activity or energy generation, which is neither realistic nor desirable. From a dynamical perspective, this point acts as an unstable source, meaning that any small increase in emissions or renewable adoption will push the system away from this state. This instability reflects the inherent momentum of industrial and energy systems; once activity begins, emissions and energy use naturally grow unless actively regulated. Thus, E_0 serves more as a theoretical baseline than a practical target, illustrating that a world entirely free of both emissions and energy use is not a stable or sustainable scenario under current socio-economic conditions.

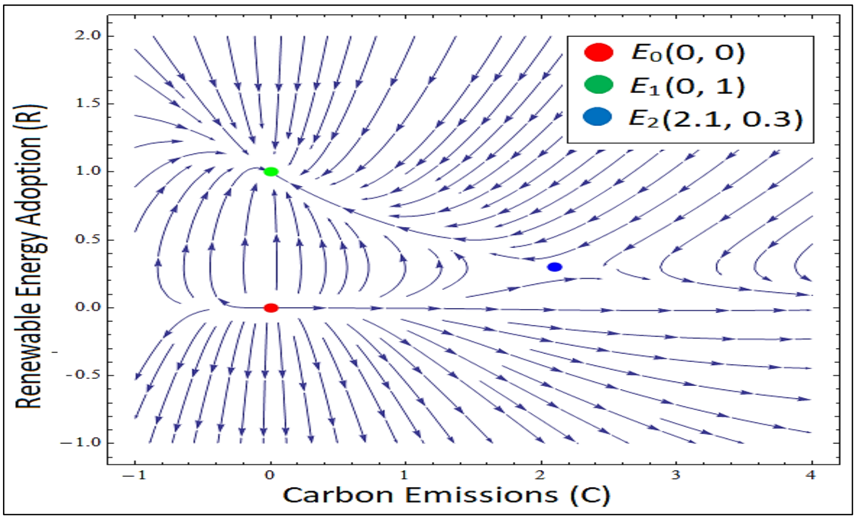

Figure 4: Phase Portrait: Carbon Emissions (C) vs. Renewable Energy Adoption (R)

The equilibrium point E_1 signifies a highly desirable state where carbon emissions have been completely eliminated and renewable energy accounts for 100% of the energy mix. This point represents the ideal outcome of a fully sustainable energy transition, with zero environmental harm from energy production. Importantly, the system identifies this point as a stable sink, meaning that trajectories in its vicinity naturally converge to it over time suggesting that, under the right conditions and momentum, the system can evolve toward full sustainability. However, reaching this point in reality would require strong, consistent policy interventions, technological breakthroughs, and global cooperation. Its stability in the model provides hope that such a transition is not only desirable but also dynamically feasible if adequately supported.

For E_2, the equilibrium values C = 2.1 and R = 0.3 represent a long-term steady state in the modeled interaction between carbon emissions and renewable energy adoption. Specifically, R = 0.3 implies that 30% of the total energy share is derived from renewable sources; an indication of moderate but not complete transition to clean energy. Meanwhile, C = 2.1, down from an initial high of 36.8, signifies a substantial reduction in carbon emissions, reflecting the positive environmental impact of increased renewable adoption. However, this equilibrium is mathematically classified as a *saddle point*, meaning it is unstable along certain directions. In practical terms, this suggests that while such a state is theoretically attainable, it is fragile and susceptible to policy, economic, or technological disruptions. Sustaining or advancing beyond this state would therefore require consistent efforts such as improving renewable efficiency, removing barriers to adoption, and maintaining strong regulatory frameworks to avoid regression toward high-emission scenarios.

Suggestions

Based on the insights obtained from the model, several policy recommendations can be proposed to support climate sustainability. Improving renewable efficiency by investing in advanced technologies, grid modernization, and energy storage can enhance the ability of renewable sources to displace carbon-intensive energy systems. Accelerating renewable energy adoption through subsidies, tax incentives, and streamlined regulatory processes can further facilitate a rapid transition toward clean energy, while strengthening infrastructure helps remove bottlenecks that slow deployment. Reducing the inhibiting effect of emissions on renewable growth requires measures that decouple fossil fuel dominance from energy markets, including carbon pricing mechanisms, divestment from fossil fuel assets, and efforts to counter misinformation that hinders clean energy transitions. It is also important to avoid emission lock-in by recognizing the instability of partial progress and maintaining long-term, consistent policy commitments that prevent regression toward high-emission trajectories. In addition, promoting active participation from both public and private sectors through inclusive frameworks involving industry, government, and civil society can ensure that renewable energy initiatives are widely adopted and equitably distributed. Collectively, these strategies aim to stabilize the energy–emission system at a desirable equilibrium and support a robust and sustainable pathway toward long-term climate goals.

Recommendations

Based on the analysis and findings of the mathematical model describing the interaction between carbon emissions and renewable energy adoption, several recommendations can be proposed to guide future actions toward achieving climate sustainability and advancing SDG 13 (Climate Action). Strengthening renewable energy infrastructure through coordinated investments by governments and the private sector can improve scalability, reliability, and accessibility, with particular emphasis on solar, wind, and hydro technologies as well as energy storage systems to support grid stability. Implementing effective emission control policies, including carbon pricing, emissions trading schemes, and fossil fuel taxation, can help internalize the environmental costs of emissions and accelerate the transition toward cleaner energy alternatives. Continued support for research and development in energy efficiency, smart grids, and next-generation renewable technologies is essential to enhance system efficiency and reduce technological and economic barriers to adoption. Increasing public awareness and participation through educational initiatives and community-level engagement can further strengthen support for renewable energy, while encouraging behavioral change and decentralized adoption such as rooftop solar systems. Maintaining long-term and consistent policy commitment is crucial, as weakening momentum may cause the system to settle into unstable equilibria, and abrupt regulatory changes can hinder progress toward sustainability. Finally, dynamic monitoring and evaluation using real-time data and system modeling can support adaptive policy responses and ensure continued alignment with long-term climate objectives.

By implementing these recommendations, policymakers and stakeholders can effectively guide the energy-emission system toward a stable, low-carbon future, avoiding unstable trajectories and reinforcing resilience in climate action efforts.

Discussion

The results obtained in this study are consistent with a growing body of literature emphasizing the nonlinear and feedback-driven nature of sustainability transitions. Previous system-dynamics and SDG-oriented studies have highlighted that progress toward climate goals is rarely linear and is often characterized by threshold effects and interaction-driven outcomes (9, 14, 15). The identification of multiple equilibria in the present model reinforces these findings, suggesting that climate–energy systems may stabilize in fundamentally different long-term states depending on policy strength and technological efficiency.

The stability of the zero-emission equilibrium aligns with earlier studies demonstrating that sustained renewable energy deployment can, under favorable conditions, drive long-term decarbonization (8, 15, 28). Conversely, the existence of an unstable intermediate equilibrium is consistent with prior observations that partial transitions may lead to fragile or reversible outcomes rather than permanent sustainability (9, 19). Such intermediate states have been discussed in the literature as manifestations of transition inertia, where economic and structural dependencies delay or obstruct full decarbonization (17, 20).

The threshold behavior observed with respect to key parameters is also supported by earlier work on SDG interactions and climate tipping points. Studies have shown that incremental improvements may have limited impact until critical thresholds are crossed, after which rapid system-wide transitions can occur (6, 14, 29). In this context, the bifurcation-like behavior identified in the model provides a simplified analytical interpretation of such tipping dynamics, complementing more complex integrated assessment and empirical approaches.

Overall, the present results support the broader consensus that achieving SDG 13 requires coordinated interventions rather than isolated policy actions. By explicitly incorporating nonlinear feedbacks, the model offers a conceptual explanation for outcomes reported in previous studies and underscores why sustained policy commitment, technological innovation, and structural change are essential for avoiding long-term climate lock-in.

Limitations and Future Research Directions

While the proposed nonlinear model provides qualitative insights into the interaction between carbon emissions and renewable energy adoption, several limitations should be acknowledged. First, the model is intentionally low-dimensional and conceptual, focusing on aggregate dynamics rather than sector-specific or regional variations. As a result, it does not explicitly account for differences across industries, geographic regions, or energy technologies, which may influence transition pathways in practice.

Second, the parameter values used in the numerical simulations are representative rather than fully calibrated to empirical datasets. Although they reflect plausible real-world magnitudes, precise estimation and validation using historical emission and energy data would improve the model's predictive capability. Additionally, the model assumes constant parameters over time, whereas real-world policy, technological progress, and economic conditions evolve dynamically.

Third, the framework does not include stochastic disturbances, time delays, or spatial effects, all of which are known to influence climate–energy systems. Random shocks such as economic crises, technological breakthroughs, or abrupt policy changes may alter transition dynamics in ways not captured by a deterministic formulation.

Future research may extend this work by incorporating data-driven parameter estimation, control variables representing policy interventions, and time-dependent or stochastic parameters. Higher-dimensional models including additional sustainability indicators, such as economic growth or energy demand, could also be explored. Furthermore, coupling the present framework with empirical data or integrated assessment models would enhance its relevance for decision-making and long-term climate planning.

Conclusion

This study presents a mathematical framework that elucidates the nonlinear co-evolution between carbon emissions and renewable energy adoption. Through equilibrium and stability analysis, three distinct long-term states were identified, with only the full renewable adoption and zero-emission equilibrium proving stable. The findings indicate that natural system dynamics do not inherently ensure sustainability without deliberate intervention. Sustained policy support, economic incentives, and technological innovation are essential to guide the system toward a clean energy equilibrium. The model offers a theoretical foundation for understanding energy–emission interactions and provides a basis for developing effective climate sustainability strategies.

Abbreviations

COVID: Coronavirus Disease, ICT: Information and Communications Technology, R&D: Research and Development, SDG: Sustainable and Development Goal, SWM: Solid Waste Management, VPA: Violence Prevention Alliance, WHO: World Health Organization, WHOW-KG: Water Health Open Knowledge Graph.

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Author Contributions

All authors contributed equally.

Conflict of Interest

The authors declare no conflict of interest.

Declaration of Artificial Intelligence (AI) Assistances

During the preparation of this work, the authors used ChatGPT by OpenAI to rephrase a few background sentences for clarity. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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