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Mental Construction Processes and Students' Errors in Solving Green's Theorem Problems

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Abstract

A deep understanding of Green's Theorem (GT) requires students to construct coherent mental representations that connect line integrals and double integrals within a unified conceptual framework. However, many students struggle to develop such integrated understanding when solving advanced calculus problems. This study aims to analyze students' mental construction processes and identify the types of errors they make when solving GT problems. A descriptive qualitative approach was employed involving 23 fourth-semester mathematics students from the FMIPA, Universitas Negeri Padang. Data were collected through written tests, semi-structured interviews and documentation to capture students' reasoning and problem-solving processes. The analysis of mental construction was guided by APOS (Action–Process–Object–Schema) theory, while students' errors were examined using the Newman's Error Analysis (NEA) framework. Although most students were able to perform actions such as recalling formulas and identifying given information, many encountered difficulties at the process and object stages, particularly in transforming line integrals into double integrals and determining the correct orientation of the curve. In addition, the most dominant errors were transformation errors and process skill errors, reflecting students' difficulties in translating problems into appropriate mathematical representations and executing solution procedures accurately. The study highlights the need for instructional strategies that emphasize conceptual mental construction, visual reasoning and reflective error analysis to enhance students' conceptual understanding and problem-solving competence in advanced calculus.

Keywords: APOS Theory, Green's Theorem, Mental Construction, Newman's Error Analysis.

Introduction

Calculus plays a central role in science, engineering and economics as a mathematical foundation for modeling change and solving complex problems (1, 2). In physics and engineer-ing, calculus is used to analyze motion, design mechanical systems and develop algorithms, while in economics it supports the analysis of price dynamics, profit optimization and production costs (3, 4). Consequently, a solid understanding of calculus enhances students' analytical thinking and problem-solving abilities. However, learning calculus requires mastery of abstract concepts such as limits, derivatives and integrals, which demand logical and systematic reasoning (5, 6).

Within vector calculus, line integrals constitute a fundamental topic and serve as a prerequisite for understanding integral theorems, including GT (7). GT connects line integrals and double integrals, requiring students to coordinate multiple representations and concepts simultaneously. Students who lack proficiency in basic integration techniques often encounter difficulties in determining curve orientation, identifying integra-tion boundaries, transforming line integrals into double integrals and performing accurate algebraic operations. A study reported that students frequently struggle to select appropriate solution methods for integral problems (8), despite the fact that mastery of integration techniques is essential in advanced calculus courses (9).

Errors in solving integral problems and integral theorems are commonly associated with weak foundational skills and a lack of procedural accuracy, leading to technical and systematic errors (10). When such errors persist, students' confidence and motivation in learning mathematics tend to decline (8). Therefore, investigating students' mental construction processes in solving GT problems is crucial for identifying the cognitive sources of these difficulties and for informing instructional design (11). Learning integrals remains challenging for both students and instructors (12, 13) due to the

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abstract nature of the concept and its strong interconnections with functions, limits, series and derivatives (14–16). Students often interpret integrals narrowly as the area under a curve or as the inverse of differentiation, resulting in fragmented conceptual understanding (17). This limited perspective contributes to high levels of anxiety and negative attitudes toward calculus learning (18). A study further emphasized that students frequently lose confidence after calculus courses due to the heavy reliance on theorems, formal procedures and interdisciplinary reasoning (19).

From a cognitive perspective, understanding GT requires students to simultaneously coordinate geometric visualization, vector field interpretation, boundary orientation, symbolic manipulation and transformations between line integrals and double integrals. Such coordination reflects a highly complex cognitive activity because students must integrate multiple representations within a unified mathematical structure. Consequently, GT possesses substantial intrinsic cognitive complexity, particularly for students whose prerequisite understanding of limits, derivatives and integrals remains fragmented. These cognitive demands may impose high intrinsic cognitive load, making it difficult for students to organize and connect mathematical ideas coherently during problem solving.

The present study adopts APOS theory as the primary theoretical framework for examining students' mental construction processes. APOS theory, which originates from constructivist learning theory, posits that mathematical understanding is actively constructed through reflective abstraction and cognitive restructuring rather than passively received from instruction. Within this framework, students construct mathematical knowledge through four interconnected mental structures. In the context of GT, mental construction involves several important cognitive processes, including interiorization of procedural actions, coordination between graphical and symbolic representations, encapsulation of mathematical processes into conceptual objects and schema formation through the integration of multiple concepts into coherent problem-solving structures. These processes enable students to gradually transform procedural experiences into deeper conceptual understanding.

Previous studies have documented various student errors in mathematics learning, including computational inaccuracies and deeper conceptual misunderstandings. A study found that procedural and calculation errors were more prevalent than conceptual errors in algebraic operations (20). Misconceptions often arise from difficulties in interpreting mathematical symbols, reasoning errors and incorrect graphical representations (21). Furthermore, NEA has been shown to effectively identify students' difficulties in problem solving, particularly in understanding and transforming mathematical problems (22). In advanced calculus contexts, such errors may indicate procedural deficiencies and incomplete cognitive construction of mathematical concepts.

Despite the extensive body of research on students' understanding of calculus and the widespread application of APOS theory to topics such as limits, derivatives, integrals and double integrals (15, 23, 24), empirical studies that explicitly examine students' mental construction of GT remain scarce. Existing research tends to focus either on procedural difficulties in integral computation or on isolated conceptual understanding, without integrating a structured cognitive framework with a systematic analysis of problem-solving errors. In particular, little attention has been given to how students' APOS mental structures relate to specific types of errors as classified by NEA when solving GT problems. This gap limits a comprehensive understanding of how conceptual development and error patterns interact in vector calculus learning.

To address this gap, the present study adopts APOS theory to analyze students' mental construction processes in solving GT problems and employs NEA to identify the types of errors students make. By integrating cognitive analysis and error analysis, this study seeks to provide deeper insights into students' conceptual and procedural understanding of GT, particularly regarding how students construct, coordinate and apply mathematical ideas during problem solving. Furthermore, this study is expected to contribute theoretically to the understanding of cognitive development in advanced calculus learning and pedagogically to the design of more effective instructional strategies in vector calculus education.

Methodology

Research Design

This study employed a qualitative descriptive approach grounded in APOS theory to examine students' mental construction processes and error patterns in solving GT problems. A Genetic Decomposition (GD) was designed to describe the expected development of students' understanding of GT, particularly the coordination between line integrals and double integrals (25). The GD served as an analytical framework to identify students' progression through the APOS stages, as shown in Figure 1.

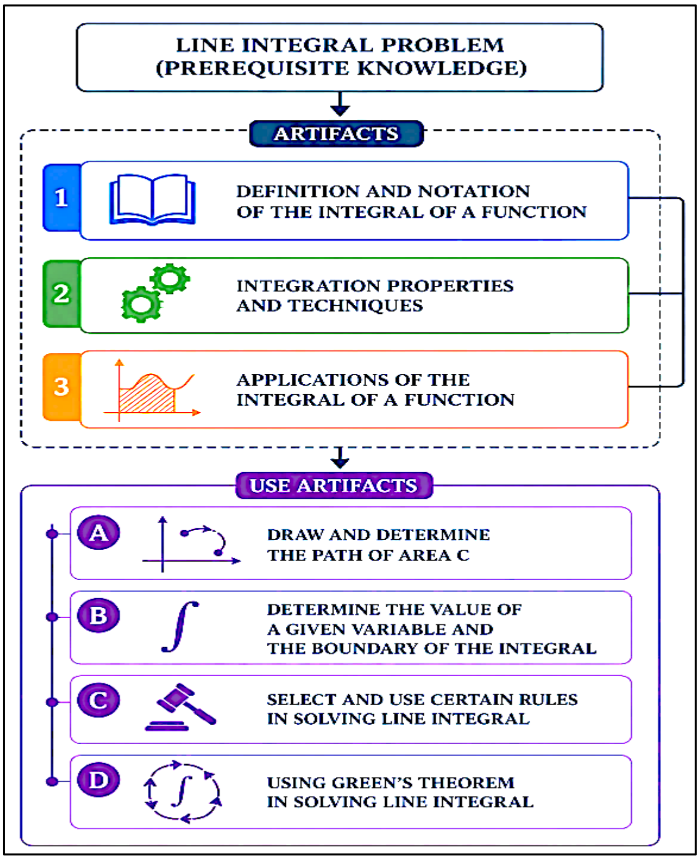

Figure 1: Genetic Decomposition for Line Integral Topic

Figure 1 presents the sequence of conceptual development expected during students' learning of line integrals and GT. The upper section of the figure represents prerequisite knowledge, including the definition and notation of integrals, integration properties and techniques and applications of integrals. These prerequisite concepts function as foundational cognitive structures that support students' later understanding of vector calculus concepts.

Furthermore, Figure 1 also illustrates how students are expected to use these prerequisite concepts when solving GT problems. The lower section describes a sequence of mathematical activities, beginning with graphical representation of regions, followed by determining variable boundaries, selecting appropriate integration techniques and finally applying GT in problem solving. This progression reflects the transition from procedural engagement toward conceptual integration within the APOS framework.

The present study specifically examined students' mental construction through four cognitive components derived from APOS theory, namely interiorization, coordination, encapsulation and schema formation. Interiorization referred to students' ability to internalize procedural actions into mental processes without relying solely on explicit instructions. Coordination involved students' ability to connect graphical, symbolic and analytical representations when solving GT problems. Encapsulation described the transformation of procedural processes into coherent mathematical objects, particularly when students conceptualized GT as an integrated relationship between line integrals and double integrals. Schema formation referred to students' ability to organize multiple mathematical concepts into a unified and flexible problem-solving structure. These cognitive indicators were used as analytical references in interpreting students' written responses and interview data.

To distinguish the nature of students' difficulties more precisely, this study differentiated between conceptual confusion and procedural deficiency. Conceptual confusion was identified when students demonstrated incorrect understanding of mathematical concepts, misinterpreted the meaning of regions or boundary orientations, or failed to construct appropriate analytical representations from geometric information. In contrast, procedural deficiency referred to inaccuracies occurring during algebraic manipulation, arithmetic operations, or symbolic computation despite the use of appropriate concepts or strategies. This distinction was considered important because conceptual and procedural difficulties reflect different levels of cognitive construction and require different pedagogical interventions.

Participants and Learning Implementation

The study involved 23 undergraduate students who were enrolled in a vector calculus course. Instruction was conducted using the Activity–Class Discussion–Exercises (ACE) cycle, which is designed to support the development of APOS mental structures through iterative and constructive learning experiences.

In the activity stage, students engaged in exploratory talks aimed at providing concrete experiences related to GT. These activities corresponded to the action stage, where students followed explicit procedures based on given instructions. Tasks included symbolic manipulation, graphical exploration of regions and the use of mathematical software to compute line integrals or double integrals numerically. The purpose of this stage was to familiarize students with procedural aspects before deeper conceptual internalization.

The class discussion stage emphasized social interaction and reflection. Students discussed their activity results collaboratively under lecturer facilitation. This stage supported the transition from action to process, as students began to internalize procedures and explain solution steps independently. Through guided questioning and multiple representations, discussions also facilitated the emergence of the object stage, in which GT was understood as a coherent mathematical entity rather than a collection of procedures.

The exercise stage consisted of structured problem-solving tasks of increasing complexity designed to reinforce and integrate students' understanding. This stage supported the formation of schemas, enabling students to coordinate concepts related to line integrals, double integrals and region orientation within a unified framework. One of the test items administered to students is defined by Equation [1]:

$$\text{Find: } \oint_C (x^2 - xy^3)dx + (y^2 - 2xy)dy \qquad [1]$$

Where, C is the area of the triangle with angles [0, 0], [2, 4] and [2, 0] using GT.

Data Collection and Analysis

Students' written responses were assessed using a maximum score of 15 and classified according to APOS stages based on their demonstrated understanding. In addition, students' errors were analyzed using NEA. Student responses were examined to identify five types of errors: Reading Error (RE), Comprehension Error (CE), Transformation Error (TE), Process Skill Error (PE) and Encoding Error (EE). RE occurred when students failed to interpret symbols or terms correctly. CE was identified when students were unable to state what was known or required in the problem. TE referred to the use of incorrect concepts or inappropriate solution strategies. PE involved computational or algebraic inaccuracies, while EE resulted from incorrect or incomplete final answers (26–28). These errors were considered influential in determining students' overall performance (2, 29). The criteria for categorizing student responses based on NEA are presented in Table 1.

Table 1: Classification of Student Errors in Solving GT

Student's Answer	NEA
No answer	RE
Incorrectly draw and determine the trajectory in region C	CE
Incorrect in determining the line equation and integral boundary	TE
Incorrect integration and algebraic calculation operation	PE
An incorrect conclusion	EE

Note: GT = Green's Theorem, NEA = Newman's Error Analysis, RE = Reading Error, CE = Comprehension Error, TE = Transformation Error, PE = Process Skill Error, EE = Encoding Error.

As shown in Table 1, RE occurred when students failed to provide a response or incorrectly interpreted mathematical symbols and information presented in the problem. CE referred to students' inability to represent the region or determine the trajectory correctly, indicating incomplete understanding of the problem context. TE and PE were associated with conceptual and procedural difficulties, particularly in determining integration boundaries, constructing line equations and performing algebraic operations accurately. EE appeared when students reached the final stage of the solution process but produced incorrect conclusions despite completing earlier procedures.

Representative excerpts of students' work were randomly selected to illustrate each error type and to provide deeper insight into students' cognitive constructions during GT problem solving. The classification criteria in Table 1 also served as the basis for connecting students' error patterns with their APOS mental structures during data analysis. The coding process for APOS analysis was conducted iteratively using indicators adapted from the theoretical characteristics of action, process, object and schema constructions. Students were categorized at the action stage when they demonstrated formula recall, direct procedural execution, or identification of known information without conceptual explanation. The process stage was identified when students were able to construct coordinate systems, interpret regions and explain procedural steps independently. Students were classified at the object stage when they successfully transformed line integrals into double integrals, determined appropriate boundaries and orientations and conceptualized GT as an interconnected Mathematical structure. Finally, students were considered to have reached the schema stage when they integrated multiple concepts, representations and solution strategies coherently and flexibly during problem solving.

To enhance the trustworthiness of the analysis, triangulation was conducted through the integration of written test results, interview transcripts and classroom observations. The triangulation process enabled the researchers to compare students' written procedures with their verbal explanations and observed reasoning processes during learning activities. Furthermore, inter-rater validation was employed to ensure consistency in APOS coding and error classification. Two mathematics education researchers independently reviewed students' responses and categorized them according to APOS stages and NEA. Any discrepancies in coding were discussed until consensus was reached, thereby improving the credibility and dependability of the findings.

To triangulate the finding's, semi-structured interviews were conducted with three students representing high, medium and low achievement levels. Participants were selected through purposive sampling based on their test scores and active participation during the learning process. The interviews aimed to clarify students' written responses and to explore their cognitive reasoning when solving GT problems, particularly in dividing regions and determining orientations. Interview data were analyzed using thematic analysis procedures consisting of data reduction, data display and conclusion drawing.

Results

During the activity stage, students were introduced to problems involving the computation of line integrals as a prerequisite to applying GT. Students engaged in exploratory tasks such as sketching regions, identifying boundary curves and performing symbolic manipulation, either manually or with the assistance of mathematical software. These activities were designed to support the action stage, where students followed explicit instructions to perform procedural steps without yet demonstrating conceptual integration.

The class discussion stage facilitated the transition from action to process. Through guided discussions, students examined the relationship between line integrals and physical interpretations such as work done by a force field, as well as the distinction between scalar and vector line integrals. At this stage, students began to internalize procedural steps into mental processes that could be executed independently of direct instruction. Discussions also supported the emergence of the object stage, in which students started to conceptualize line integrals as abstract mathematical entities associated with specific trajectories and regions. Lecturers employed reflective questioning and graphical representtations to reinforce this conceptual shift.

In the exercise stage, students worked on structured problems intended to consolidate and organize their understanding into coherent schemas. These exercises required students to analyze regions bounded by piecewise curves, determine appropriate orientations and select suitable solution strategies, including the application of GT. Through these tasks, students were expected to integrate multiple concepts, such as line integrals, double integrals and region orientation, into a unified problem-solving framework. Based on students' written responses, indicators for each APOS stage were identified.

a) Students who were able to recognize and restate the problem context were categorized at the action stage.
b) Students who correctly constructed Cartesian coordinates and sketched the region C accurately were classified at the process stage.
c) At the object stage, students were able to determine the trajectory of region C, formulate the equation of the boundary curves, identify appropriate integral limits and apply relevant integration rules.
d) Students were considered to have reached the schema stage if they applied GT explicitly to transform the line integral into a double integral.

Observable cognitive indicators also emerged across the APOS stages. Students at the action stage primarily demonstrated formula recall and direct symbolic substitution without conceptual explanation. At the process stage, observable signs included coordinate construction, graphical representation of regions and procedural explanation of solution steps. Students categorized at the object stage showed the ability to perform integral transformations and coordinate geometric interpretations with analytical expressions.

Meanwhile, students who reached the schema stage demonstrated conceptual integration through flexible reasoning, coherent boundary analysis and systematic coordination between graphical, symbolic and computational representations. The transition from process to object stage reflected students' mathematical abstraction, where procedures were no longer viewed merely as operational steps but as coherent conceptual entities.

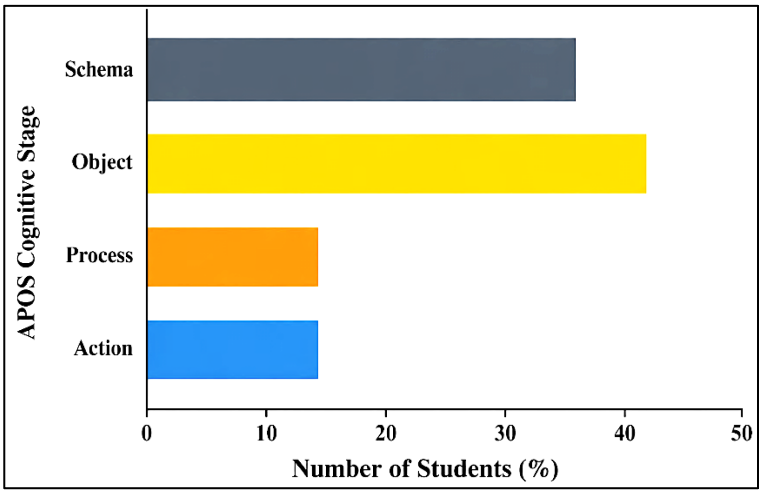

Figure 2: Students' Achievement in Solving GT

Figure 2 illustrates the distribution of students' cognitive achievement levels in solving GT problems based on APOS theory. Of the total participants, three students were classified at the action stage, indicating that they were only able to engage with the problem at a procedural level, primarily through interpreting symbols and following explicit instructions. Three students reached the process stage, demonstrating the ability to construct Cartesian coordinates and represent the region correctly, yet still showing limited internalization of procedures and conceptual understanding.

The largest group, consisting of nine students, achieved the object stage. These students were generally able to identify boundary curves, formulate equations of trajectories and determine integration limits, suggesting that they had begun to conceptualize GT and line integrals as coherent mathematical objects. However, as reflected in earlier error analysis, some students at this stage still encountered difficulties in transforming geometric representations into correct analytical forms.

Finally, eight students reached the schema stage, indicating their ability to integrate multiple concepts, such as region orientation, boundary decomposition and integral computation, into a unified problem-solving framework. Students at this level demonstrated a more holistic understanding of GT, although minor procedural or representational inaccuracies were still observed in some cases.

Table 2: Relation of APOS Stages to NEA

APOS Stages	NEA				
	RE	CE	TE	PE	EE
Action	8.70%	-	-	-	-
Process	-	4.35%	-	-	-
Object	-	-	13.04%	-	-
Schema	-	-	-	39.13%	4.35%

Note: APOS= Action–Process–Object–Schema, NEA = Newman's Error Analysis, RE = Reading Error, CE = Comprehension Error, TE = Transformation Error, PE = Process Skill Error, EE = Encoding Error.

Table 2 presents the distribution of students' responses across APOS stages in relation to NEA. At the action stage, 8.70% of students exhibited RE, indicating difficulties in interpreting symbols, notation, or the overall meaning of the GT problem. This suggests that a small proportion of students were unable to move beyond surface-level engagement with the problem statement. At the process stage, 4.35% of students committed CE, reflecting an inability to correctly identify what was known and what was required, particularly in constructing the region and understanding the role of boundary orientation. These findings imply that although students could initiate procedural steps, they lacked a coherent understanding of the underlying mathematical relationships.

At the object stage, TE emerged in 13.04% of students' responses. Students at this stage generally succeeded in visualizing the region and identifying boundary curves; however, they failed to correctly transform the line integral into an appropriate mathematical form, such as determining correct parameterizations, substitutions, or integration limits. This pattern indicates that students had partially constructed the concept of GT as an object but had not yet achieved conceptual flexibility in applying it accurately.

Interestingly, at the schema stage, errors shifted toward PE and EE, with proportions of 39.13% and 4.35%, respectively. Although students at this stage attempted to integrate multiple concepts and apply GT within a broader framework, inaccuracies in algebraic manipulation, integral computation and final expression of results were still evident. This suggests that reaching the schema stage does not guarantee error-free performance, as procedural fluency and accuracy remain critical components of successful problem solving. To clarify the distribution of students' difficulties more explicitly, Table 3 summarizes the dominant error categories identified in this study.

Table 3: Distribution of Students' Errors in Solving GT Problems

Error Type	Percentage	Dominant APOS Stage	Example
RE	8.70	Action	Misinterpretation of notation.
CE	4.35	Process	Incorrect region representation.
TE	39.13	Object	Incorrect boundary and integration limits.
PE	30.43	Schema	Algebraic manipulation inaccuracies.
EE	4.35	Schema	Incorrect final conclusion.

Note: APOS= Action–Process–Object–Schema, RE = Reading Error, CE = Comprehension Error, TE = Transformation Error, PE = Process Skill Error, EE = Encoding Error.

The findings in Table 3 indicate that TE was dominant at the object stage, where students had begun to conceptualize GT but still experienced substantial difficulty transforming geometric representations into correct analytical formulations. In contrast, PE was more dominant at the schema stage, suggesting that students who had achieved broader conceptual integration still encountered difficulties maintaining procedural precision during algebraic computation and integration processes. These results demonstrate that students' cognitive difficulties evolved from interpretative and representational issues toward computational and procedural inaccuracies as their APOS constructions developed.

Comparative Analysis of Student Errors

The comparison between high- and low-performing students revealed substantial differences in conceptual organization and reasoning flexibility. High-performing students demonstrated conceptual integration, coherent graphical interpretation and flexible reasoning when transforming line integrals into double integrals. In contrast, low-performing students relied predominantly on formula memorization and rigid procedural execution without fully understanding the relationships between region orientation, boundary parameterization and integration limits. High-performing students were generally able to justify their analytical decisions mathematically, whereas low-performing students tended to imitate previously learned examples without adapting them appropriately to new problem situations.

Differences also emerged between conceptual and procedural errors. Conceptual errors were primarily associated with incorrect interpretation of regions, misunderstanding of boundary orientation and failure to establish correct geometric–analytic relationships. Procedural errors, meanwhile, involved algebraic inaccuracies, arithmetic mistakes and incorrect symbolic manipulation despite the use of appropriate conceptual strategies. This distinction indicates that some students possessed partial conceptual understanding but lacked procedural fluency, while others failed to construct the underlying mathematical concepts altogether.

Error patterns additionally varied across APOS stages. At the action stage, difficulties were dominated by formula recall and symbol interpretation. At the process stage, students frequently encountered difficulties in graphical representation and coordinate construction. The object stage was characterized mainly by TE related to integral formulation and boundary determination, while the schema stage showed greater emphasis on algebraic precision and computational consistency. These findings suggest that students' cognitive difficulties evolve progressively alongside the development of their mental constructions.

A further distinction was observed between computational and interpretive difficulties. Computational difficulties included arithmetic inaccuracies, algebraic simplification errors and incorrect integration procedures. Interpretive difficulties, however, involved deeper conceptual problems such as misunderstanding orientation, region decomposition and the relationship between line integrals and area integrals. The predominance of interpretive difficulties among medium- and low-performing students indicates that many students experienced incomplete conceptual abstraction rather than merely procedural weakness.

Student 1

Student 1 began by correctly identifying the components of the vector field, namely Equations [2, 3].

$$M(x,y) = x^2 - xy^3 \quad [2]$$
$$N(x,y) = y^2 - 2xy \quad [3]$$

The student then accurately computed the partial derivatives (Equations [4, 5]) demonstrating a clear understanding of the analytical requirements of GT as shown in Figure 3.

$$\frac{\partial N}{\partial x} = -2y \quad [4]$$

$$\frac{\partial M}{\partial y} = -3xy^2 \quad [5]$$

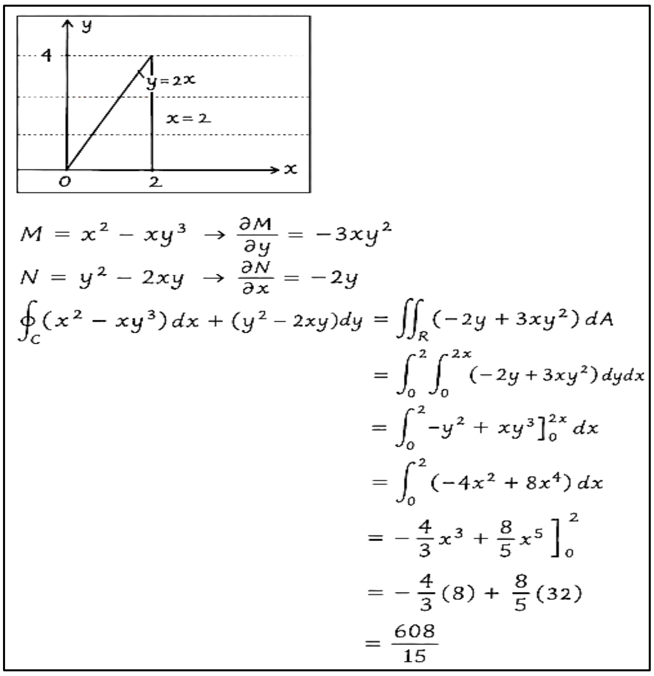

Figure 3: Representation for Answer of Student 1

This step indicates that the student had conceptualized the theorem as an object, recognizing the relationship between the line integral and the corresponding double integral over the region.

Next, the student correctly applied GT by transforming the line integral into a double integral of the form $\iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$. The region of integration was appropriately represented through a sketch and the student determined suitable integration bounds based on the geometry of the region. The order of integration was chosen consistently and substitutions were carried out systematically, reflecting a well-organized problem-solving strategy.

Table 4: Excerpts from the Researcher's Interview with Student 1

	Questions
Researcher	Could you explain why you chose to use GT to solve this problem?
Student 1	Because the line integral is taken over a closed curve and the vector field satisfies the required conditions, I considered GT to be the most efficient approach to transform the line integral into a double integral over the enclosed region.
Researcher	How did you determine the integrand for the double integral?
Student 1	I first identified the vector field components as $M(x,y)$ and $N(x,y)$. Then I computed $\frac{\partial N}{\partial x}$ and $\frac{\partial M}{\partial y}$. The difference between these two partial derivatives directly forms the integrand according to GT.

Researcher	What considerations did you make when determining the region and the limits of integration?
Student 1	I sketched the region to clearly see its shape and orientation. From that sketch, I determined appropriate bounds for x and y so that the integral fully represents the entire region enclosed by the curve.

From Table 4, the statement "I sketched the region to clearly see its shape and orientation" indicates that Student 1 coordinated graphical and analytical representations simultaneously. From an APOS perspective, this reflects schema-level understanding because the student no longer relied solely on procedural recall but demonstrated conceptual integration between geometry, orientation and integral transformation. The student's reasoning also shows evidence of encapsulation, where the transformation process was understood as a coherent mathematical object rather than isolated procedural steps. Throughout the solution, the student demonstrated strong procedural fluency in evaluating the integrals and performing algebraic manipulations. The final result was obtained through a sequence of logically connected steps, showing coherence between graphical representation, analytical transformation and computation. Overall, this response reflects schema-level understanding, as the student successfully integrated multiple concepts into a unified and mathematically sound solution of GT.

Student 2

Student 2 correctly identified the vector field components (Equations [6, 7]) and computed the partial derivatives (Equations [8, 9]).

$$M(x, y) = x^2 - xy^3 \quad [6]$$

$$N(x, y) = y^2 - 2xy \quad [7]$$

$$\frac{\partial N}{\partial x} = -2y \quad [8]$$

$$\frac{\partial M}{\partial y} = -3xy^2 \quad [9]$$

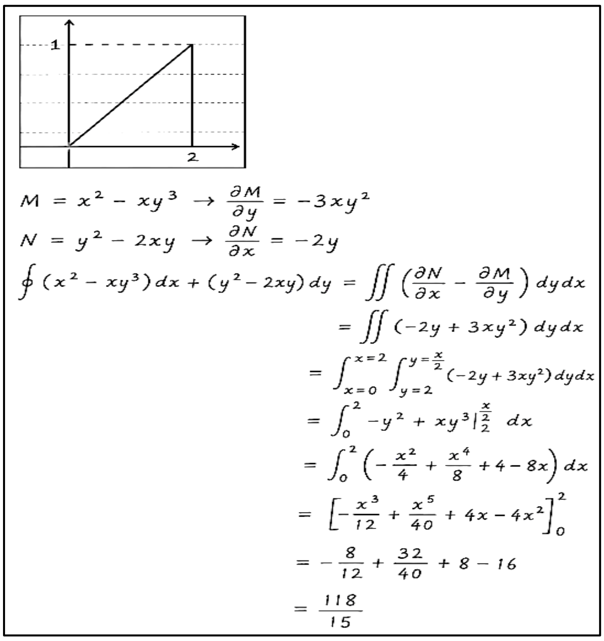

Figure 4: Representation for Answer of Student 2

This indicates that Student 2 recognized the formal structure of GT and attempted to apply it by transforming the line integral into a double integral. The student also sketched the integration region and selected an order of integration, as shown in Figure 4. However, inaccuracies emerged in determining the integration bounds and in expressing the integrand after applying $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$. These inaccuracies led to inconsistent substitutions and incorrect limits in subsequent steps. While the student proceeded with the computational process and completed the algebraic manipulations, the final result was affected by earlier conceptual misalignment.

Table 5: Excerpts from the Researcher's Interview with Student 2

	Questions
Researcher	How did you determine the integrand in the double integral?
Student 2	I followed the formula by finding $\frac{\partial N}{\partial x}$ and $\frac{\partial M}{\partial y}$, then subtracting them. I was confident about that part, but I was less certain when setting up the limits.
Researcher	What difficulties did you experience when determining the region and the limits of integration?
Student 2	I could picture the region but translating that picture into exact bounds was confusing. I wasn't completely sure which values of x and y should come first in the integration.
Researcher	How do you understand GT conceptually?
Student 2	I understand that it connects a line integral to an area integral, but I still rely heavily on the formula. If the region is complicated, I get unsure about how to set everything up correctly.

From Table 5, the phrase "I could picture the region but translating that picture into exact bounds was confusing" provides important cognitive evidence that Student 2 had partially achieved process coordination but had not fully encapsulated the transformation into an integrated conceptual object. Although the student understood the formal structure of GT, the inability to consistently convert graphical representation into analytical boundaries indicates incomplete mathematical abstraction. This response reflects object-stage understanding with dominant TE associated with geometric–analytic mapping difficulties.

Overall, Student 2 demonstrated partial conceptual understanding characteristic of the object stage in APOS theory: the student viewed GT as an applicable mathematical object and followed its formal procedure, yet lacked sufficient conceptual control over region description and boundary consistency. The dominant difficulty aligns with a TE, where the transition from geometric representation to correct analytical formulation was not fully achieved.

Student 3

Figure 5 illustrates the written solution of Student 3 in solving a GT problem. The student began by correctly identifying the vector field components (Equations [10, 11]) and computing the partial derivatives (Equations [12, 13]).

$$M(x, y) = x^2 - xy^3 \quad [10]$$

$$N(x, y) = y^2 - 2xy \quad [11]$$

$$\frac{\partial M}{\partial y} = -3xy^2 \quad [12]$$

$$\frac{\partial N}{\partial x} = -2y \quad [13]$$

$$M_x = (x^2 - xy^3) \rightarrow \frac{\partial M}{\partial y} = -3xy^2$$
$$N_x = (y^2 - 2xy) \rightarrow \frac{\partial N}{\partial x} = -2y$$
$$\oint (x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy$$
$$= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dy\,dx$$
$$= \int_0^2 \int_2^4 (-3xy^2 - 2y)\,dy\,dx$$
$$= \int_0^2 -xy^3 - y^2]_2^4\,dx$$
$$= \int_0^2 (-60x - 12)\,dx$$
$$= -30x^2 - 12x]_0^2$$
$$= -120 - 24$$
$$= -144$$

Figure 5: Representation for Answer of Student 3

This indicates that Student 3 was able to recall and apply the formal formula of GT at a procedural level.

However, substantial difficulties emerged in the subsequent steps. The student attempted to apply GT by forming a double integral of $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$, yet showed confusion in simplifying the integrand, leading to incorrect algebraic expressions. In addition, the determination of integration boundary was inconsistent and not clearly derived from a well-defined region. The region itself was not explicitly represented or justified through a sketch, which weakened the connection between the geometric interpretation and the analytical formulation.

Table 6: Excerpts from the Researcher's Interview with Student 3

	Questions
Researcher	How did you determine the region and the boundary of integration?
Student 3	I wasn't very sure about the region. I tried to follow the example from class and chose limits that I thought matched the problem.
Researcher	What part of the solution did you find most difficult?
Student 3	I found it hard to connect the picture of the region with the integral. I knew the formula, but I didn't really understand how the limits should come from the region.
Researcher	How would you describe your understanding of GT?
Student 3	I know the steps and the formula, but I don't fully understand why it works or how the line integral and the double integral are related.

From Table 6, the statement "I knew the formula, but I didn't really understand how the limits should come from the region" strongly indicates reliance on procedural memorization without conceptual interiorization. From an APOS perspective, Student 3 remained primarily at the action stage because the student executed symbolic procedures mechanically without constructing coherent relationships between graphical representation and analytical formulation. The response also demonstrates incomplete coordination between geometric interpretation and integral transformation, which resulted in dominant TE and unsupported analytical assumptions.

Although the student completed the integration process and obtained a final numerical result, the solution contained multiple conceptual and procedural inaccuracies originating from incorrect transformations and unsupported assumptions. These errors indicate that Student 3 relied heavily on memorized procedures without a clear understanding of the underlying concepts. From the perspective of APOS theory, this response reflects action-level understanding, where students can follow symbolic rules but have not internalized the process or conceptualized GT as an object. The dominant error type corresponds to TE, as the student failed to correctly translate the problem from its geometric form into an appropriate analytical representation.

Discussion

Calculus remains one of the most frequently discussed topics in undergraduate mathematics, particularly derivatives and integrals (30). Geometrically, integrals are commonly introduced as the area under a curve f(x) and conceptually they play a crucial role in calculating areas, volumes, arc lengths, fluid forces and other quantities that are fundamental in physics, statistics and engineering (31). Mastery of integral concepts requires an understanding of antiderivatives and the relationship between definite and indefinite integrals (32). Despite their importance, numerous studies have reported that students' performance in solving integral-related problems remains relatively low (33), largely because students tend to prioritize procedural techniques over conceptual understanding (34).

The findings of this study reinforce previous research indicating that teaching mathematical concepts through memorization and formula recall contributes to students' misconceptions and fragmented understanding (35). As reported by Rahim, students often struggle to articulate an initial strategy for problem solving; even when an initial idea exists, they experience difficulty progressing to subsequent steps (36). Another study similarly observed that although students are frequently able to execute integration procedures, their understanding of the relationship between definite integrals, area and fundamental theorems of calculus remains limited (34). These tendencies were also evident in this study, particularly among students who relied heavily on formal rules without fully understanding the underlying concepts of GT.

Understanding GT requires strong prerequisite knowledge of limits, derivatives and integrals (30). However, students often fail to conceptualize integrals as limits of sums (17), which weakens their ability to interpret integral theorems meaningfully. Karama reported that students experience difficulties in determining derivatives using limit definitions and in interpreting graphical representations of derivatives (37), largely due to insufficient understanding of prerequisite concepts such as functions, limits and gradients (15). In the context of this study, such conceptual gaps help explain why some students struggled to connect the geometric interpretation of regions with the analytical formulation required by GT.

From a mathematical perspective, solving GT problems requires students to coordinate several interconnected analytical processes simultaneously. Students must correctly determine the orientation of closed curves, construct suitable parameterizations of boundary segments, interpret the relationship between circulation and area integration and map geometric regions into appropriate analytical representations. Many students in this study experienced difficulties preserving the counterclockwise orientation required by GT, resulting in sign inconsistencies and incorrect integral formulations.

In several cases, students also demonstrated incomplete understanding of parameterization when expressing piecewise boundary curves analytically. These findings indicate that students often viewed the transformation from line integrals to double integrals procedurally rather than conceptually. Furthermore, although Jacobian transformation was not explicitly required in the given task, students' difficulties in converting geometric regions into analytical integral bounds suggest broader weaknesses in geometric–analytic mapping, which is fundamental in multivariable and vector calculus learning.

Differences in students' mental constructions play a significant role in their success or failure in solving mathematical problems (33, 38). APOS theory provides a useful framework for characterizing these differences by describing how students' progress through the action, process, object and schema stages (39, 40). The present study demonstrates that students' levels of understanding of GT varied considerably. While some students reached the schema stage and were able to integrate multiple concepts coherently, others remained at the object or process stages, indicating incomplete mental construction. The design of GD proved effective in revealing how students constructed or failed to construct key concepts (24, 41).

Analysis of individual student work further revealed that medium- and low-ability students exhibited incomplete understanding even when they appeared to reach the object stage. These students often failed to determine all boundary trajectories, assign correct orientations, or specify appropriate integration boundaries, resulting in incorrect solutions. This finding aligns with past research who reported that errors are more prevalent among students with medium and low mathematical ability, largely due to differences in analytical skills and problem-solving attitudes (27). Moreover, a study showed that students may demonstrate adequate understanding at the action stage but encounter difficulties at the process and object stages, particularly in explaining graphical constructions (42).

The comparison between high- and low-performing students revealed important differences in cognitive organization and problem-solving flexibility. High-performing students generally demonstrated schema-level understanding characterized by conceptual integration, flexible reasoning and the ability to justify transformations between geometric and analytical forms. In contrast, low-performing students tended to rely heavily on memorized procedures and isolated formulas without fully understanding the meaning of boundary orientation, region decomposition, or integral transformation.

Conceptual confusion was particularly evident when students incorrectly interpreted the enclosed region or selected inappropriate integration limits, whereas procedural deficiencies appeared primarily in algebraic simplification and arithmetic operations. These findings indicate that conceptual understanding and procedural fluency are interdependent rather than separate dimensions in advanced calculus learning.

The error analysis based on NEA revealed several significant difficulties experienced by students in solving GT problems. These errors manifested as failure to respond to the problem, incorrect representation of regions and trajectories and inaccurate determination of equations or integration limits. Such findings are consistent with another who reported that TE and PE were common in integral problems due to students' inability to convert one mathematical form into another and their lack of algebraic accuracy (43).

The dominance of TE in this study is also similar with past study which identified similar error patterns in students solving PISA problems, attributing them to difficulties in translating contextual problems into mathematical models (26). Notably, EE were rarely observed, which is consistent with previous findings indicating that such errors are uncommon among students who possess at least a basic level of mathematical competence (29). Prior research has consistently shown that errors in mathematical problem solving significantly affect learning outcomes (28, 44) and that misconceptions at the conceptual level led to persistent confusion in subsequent problem solving (45).

The predominance of transformation errors in this study indicates that many students experienced difficulty translating geometric representations into analytical integral forms. This suggests that the transition from process to object construction had not been fully achieved. Students were often able to identify formulas symbolically but failed to coordinate graphical interpretation, orientation analysis and boundary formulation coherently. Such findings support the view that transformation errors are strongly associated with incomplete encapsulation processes within APOS theory. Consequently, students may appear procedurally competent while still lacking conceptual control over the mathematical structures underlying GT.

Overall, the findings suggest that difficulties in solving GT problems stem not only from procedural weaknesses but also from incomplete mental constructions and conceptual misunderstandings. These results highlight the importance of instructional strategies that explicitly address students' mental construction processes and common error patterns (10,42). By incorporating APOS-based learning designs and systematically analyzing student errors, lecturers can develop more effective pedagogical approaches that minimize misconceptions, strengthen conceptual understanding and improve the overall quality of vector calculus learning.

From a pedagogical perspective, the ACE cycle can serve as an effective instructional framework for reducing students' conceptual and transformation errors in GT learning. In the activity phase, lecturers should provide exploratory talks involving graphical visualization, orientation tracing and region decomposition to strengthen students' geometric intuition before formal symbolic manipulation is introduced.

During the class discussion phase, scaffolding can be implemented through guided questioning, peer explanation and reflective comparison between correct and incorrect solution strategies. Such scaffolding is important for helping students coordinate geometric representations with analytical formulations and for facilitating the transition from process to object construction. In the exercise phase, students should be exposed progressively to increasingly complex regions and parameterizations so that they can develop flexible schemas for applying GT in different contexts.

In addition, the integration of GeoGebra may help students visualize orientations, boundary trajectories and integral transformations more meaningfully. Lecturers should also emphasize reflective error analysis by discussing common transformation and orientation errors explicitly during instruction. Through systematic scaffolding and visualization-based learning, students are expected to develop stronger conceptual understanding, improved geometric–analytic mapping ability and greater flexibility in solving advanced vector calculus problems.

Conclusion

This study demonstrates that students' understanding of GT develops through varying levels of mental construction as described in APOS theory. While several students were able to reach the schema stage and integrate geometric interpretation, boundary orientation and integral transformation coherently, many others remained at the action, process, or object stages, indicating incomplete conceptual construction. The findings reveal that successful problem solving in GT requires not only procedural fluency but also deep conceptual understanding of the relationships between line integrals, double integrals, region orientation and analytical representations. Students who relied primarily on memorized formulas without sufficient conceptual coordination tended to experience substantial difficulties in constructing mathematically valid solutions.

The results further indicate that the most dominant difficulties encountered by students were related to transformation processes, particularly in determining boundary trajectories, representing regions analytically, selecting appropriate integration limits and maintaining correct curve orientation. These difficulties were predominantly classified as TE within the NEA. Such findings suggest that students often experience challenges in translating geometric information into analytical integral forms, reflecting incomplete encapsulation and schema formation processes within APOS theory. In addition, procedural deficiencies such as algebraic inaccuracies and computational inconsistencies were also observed, especially among students who had partially achieved higher APOS stages but still lacked procedural precision. These findings confirm that conceptual understanding and procedural competence must develop simultaneously in advanced calculus learning.

From a pedagogical perspective, the findings emphasize the importance of instructional designs that explicitly support students' cognitive construction processes during vector calculus learning. The ACE cycle provides meaningful opportunities for students to gradually progress from procedural engagement toward conceptual integration through exploratory activities, reflective discussions and structured exercises. Effective scaffolding should therefore be systematically implemented at each stage of learning, particularly through graphical visualization, guided orientation analysis, reflective questioning and collaborative reasoning activities. Furthermore, lecturers are encouraged to incorporate diagnostic error analysis into classroom instruction so that students become more aware of common conceptual and transformation errors when solving GT problems. Such approaches are expected to strengthen students' geometric–analytic reasoning, reduce misconceptions and improve overall conceptual understanding in advanced calculus courses.

The study involved a relatively small number of participants from a single university context, which may limit the generalizability of the findings to broader educational settings. In addition, the study focused specifically on GT and did not investigate whether similar cognitive patterns occur in other vector calculus topics such as Stokes' Theorem or the Divergence Theorem. Therefore, future research is recommended to employ longitudinal and mixed-method approaches involving larger and more diverse participant groups, dynamic visualization environments and technology-assisted learning interventions to further explore students' mental construction processes and cognitive development in advanced mathematics learning.

Abbreviations

ACE: Activities–Class Discussion–Exercises, APOS: Action–Process–Object–Schema, CE: Comprehension Error, EE: Encoding Error, GD: Genetic Decomposition, GT: Green's Theorem, NEA: Newman's Error Analysis, PE: Process Skill Error, RE: Reading Error, TE: Transformation Error.

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Author Contributions

Kelly Angelly Hevardani: Investigation, Writing – Original Draft, Yerizon: Conceptualization, Methodology, Investigation, Writing – Original Draft, Writing – Review and Editing, Yarman: Methodology, Writing – Original Draft, Writing – Review and Editing, Armiati: Methodology, Writing – Review and Editing, Fitrani Dwina: Writing – Review and Editing.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

The data from this study are not available to the public to protect participants' confiden-tiality and comply with the institution's research policies.

Declaration of Artificial Intelligence (AI) Assistance

The authors confirm that no generative artificial intelligence tools were used to generate the scientific content, analysis, or interpretation presented in this manuscript. The authors take full responsibility for the originality, interpretation and accuracy of the content.

Ethics Approval

The data used in this study were collected as part of classroom instructional activities conducted under the Dean's assignment letter number: 7119/UN35.1/KP/2025.

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