

Designing Metacognitive Scaffolding for Mathematical Creative Thinking in Graph Theory Learning

Noviana Dini Rahmawati^{1,2}, Budi Waluya^{1*}, Mulyono¹,
Iqbal Kharisudin¹

¹Doctoral Study Program in Mathematics Education, Universitas Negeri Semarang, Semarang, Indonesia, ²Mathematics Education Study Program, Universitas PGRI Semarang, Semarang, Indonesia. *Corresponding Author's Email: budiw@mail.unnes.ac.id

Abstract

Mathematical creative thinking is an essential competency in higher education mathematics learning, especially in graph theory courses that require students to analyze structures, generate alternative solutions and construct logical arguments. One strategy that can support the development of these abilities is metacognitive scaffolding, which encourages students to plan, monitor and evaluate their thinking processes while solving problems. This study aims to design a metacognitive scaffolding model to support students' mathematical creative thinking in graph theory learning. The study employed the ADDIE development model involving analysis, design, development, implementation and evaluation stages. Participants were undergraduate students enrolled in graph theory courses at a private university in Indonesia. Data were collected through mathematical creative thinking ability test. Research findings show that the metacognitive scaffolding design developed for graph theory learning consists of five important components: understanding the problem, planning a strategy, implementing the plan, evaluating and reflecting. The integration of scaffolding in Problem Based Learning supports students in identifying problems, producing various strategies, solutions and evaluating alternative answers. The validity results show that metacognitive scaffolding in the form of Textbooks and Student Worksheets, reached the "Very Good" category in terms of suitability of content, construction and linguistic aspects. In addition, the implementation of metacognitive scaffolding provides a real positive influence compared to learning without metacognitive scaffolding. Thus, metacognitive scaffolding is suitable for application in graph theory learning.

Keywords: Mathematical Creative Thinking, Metacognitive Scaffolding, Graph Theory, Problem Based Learning.

Introduction

The development of twenty-first-century competencies has become a central goal of higher education, particularly in mathematics education. Among these competencies, mathematical creative thinking is essential because it enables students to generate diverse and original ideas, develop flexible strategies, elaborate mathematical concepts and solve complex problems effectively (1, 2). Creative thinking involves generating and manipulating ideas in unconventional ways and exploring alternative solutions (3-5). In mathematics, creativity integrates creative thinking with mathematical knowledge through understanding abstract concepts, connecting representations and applying flexible thinking to problem solving (6-7). Mathematical creativity is not regarded solely as an innate ability; rather, it can be fostered through appropriate learning experiences that encourage students to explore multiple representations,

generate alternative solution strategies and engage with non-routine or multiple-solution problems (8). Therefore, higher mathematics education should extend beyond procedural mastery to provide opportunities for students to develop and evaluate mathematical ideas creatively.

Graph theory provides a relevant context for developing mathematical creative thinking because it involves visualization, mathematical modeling, systematic exploration, conjecturing, logical reasoning and problem solving. Graph problems can also provide opportunities for students to explore different representations and solution approaches, thereby supporting creative thinking dimensions such as fluency, flexibility, originality and elaboration. However, despite these opportunities, students still experience difficulties in generating creative solutions to

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graph problems, particularly when they are required to understand complex graph concepts, explore alternative approaches and develop appropriate solution strategies (9). Students also experience difficulties in elaborating ideas, connecting concepts and evaluating their reasoning. Thus, although graph theory has strong potential for fostering creativity, existing learning practices may not sufficiently facilitate the exploratory and reflective thinking required.

Metacognitive ability is closely associated with mathematical creative thinking. Metacognition involves awareness and regulation of one's thinking through planning, monitoring and evaluation. In mathematical problem solving, these skills help students recognize problem characteristics, select appropriate strategies, monitor solutions and reflect on their reasoning. Stronger metacognitive awareness is associated with better problem-solving performance and a greater capacity to generate mathematical ideas (10). However, students may lack sufficient metacognitive skills to regulate their learning effectively (11), indicating the need for instructional support that explicitly facilitates their thinking processes.

Metacognitive scaffolding provides such support by gradually guiding students toward independent and self-regulated learning. Through guiding questions, prompts, feedback, monitoring and reflection, it helps students regulate strategies and evaluate their learning processes (12, 13). Previous studies have demonstrated positive contributions of scaffolding to higher-order thinking, problem-solving abilities and learning achievement (14). However, these studies have primarily emphasized its effectiveness rather than designing metacognitive scaffolding based on the characteristics of students' mathematical creative thinking processes in graph theory. Thus, how scaffolding can systematically address students' difficulties during creative mathematical problem solving remains insufficiently explored.

Problem Based Learning (PBL) provides an appropriate instructional context for metacognitive scaffolding, as problem-solving activities encourage students to collaboratively construct knowledge while planning, monitoring and reflecting on their learning and problem-solving processes (15). Previous studies have demonstrated that PBL contributes positively to

mathematics learning by enhancing students' problem-solving and higher-order thinking skills while promoting active and motivating learning experiences (16–23). Its phases also provide opportunities to explore alternative ideas. However, PBL does not necessarily provide explicit guidance for regulating students' thinking. Consequently, students may engage in problem solving without systematically planning, monitoring, evaluating and reflecting on their creative thinking processes.

These limitations reveal an important research gap. Despite growing attention to mathematical creativity, metacognition, scaffolding and PBL, studies that develop metacognitive scaffolding based on students' mathematical creative thinking processes and systematically integrate it into PBL for graph theory learning in higher education remain limited. Previous studies tend to examine these components separately, while existing learning practices emphasize conceptual understanding and procedural problem solving without explicitly connecting creative thinking, metacognitive regulation and PBL activities.

To address this gap, the present study develops a metacognitive scaffolding framework comprising five aspects: understanding problems, planning strategies, implementing plans, evaluation and reflection, systematically integrated into PBL activities for graph theory learning. The novelty lies in designing scaffolding based on the identified characteristics of students' mathematical creative thinking processes and embedding it within PBL rather than treating creativity, metacognition and PBL as separate components. Accordingly, the main aim is to develop metacognitive scaffolding to support mathematical creative thinking in graph theory. Specifically, the study aims to develop and validate the metacognitive scaffolding and examine the practicality and effectiveness of its implementation within PBL in improving students' mathematical creative thinking. This study is expected to provide theoretical and practical contributions to designing graph theory learning that systematically fosters mathematical creativity.

Methodology

Research Design

This research designs metacognitive scaffolding using the ADDIE (Analysis, Design, Development,

Implementation and Evaluation) development model (24). This development model has a significant influence on increasing the effectiveness of student learning and the efficiency of lecturers' teaching design in certain courses. The application of the ADDIE development model has become the gold standard for professionally created, high-quality distance education programs for teaching. This model allows these principles to be discovered and executed systematically and thoroughly.

Development Procedures

The performance analysis in this study aims to identify and clarify the basic problems faced in graph learning. In this study, the problem encountered is the existence of obstacles in students' creative thinking processes in solving graph problems, thus requiring appropriate scaffolding to stimulate students' mathematical creative thinking abilities. In addition, researchers also conducted a needs analysis in the form of determining the competencies that students must achieve.

Design

At the design stage, a strategy for the scaffolding to be developed is determined, namely metacognitive scaffolding. Identify basic competencies and design forms of learning activities that are in accordance with metacognitive scaffolding, identify indicators of competency achievement and plan the form and type of research that will be presented. After planning and designing the product to be developed.

Development

At this development stage, researchers develop metacognitive scaffolding that has been designed at the design stage. The learning media that has been produced is then reviewed by several validators as media experts and material experts in order to receive input or improvements before being applied in learning. Quantitative data obtained from filling out validation sheets from learning experts was analyzed using a Likert scale in the table below for 5 scales. Table 1 presents the Likert scale used to assess the questionnaire responses.

Table 1: Likert Scale Level

Statement	Positive	Negative
Strongly agree	5	1
Agree	4	2
Doubt	3	3
Don't agree	2	4
Strongly Disagree	1	5

After the questionnaire is validated by the validator, the questionnaire is then analyzed and presented as a percentage. The percentage was calculated using the Equation [1]:

$$\text{Percentage (P)} = \frac{\text{Total score obtained}}{\text{Maximum Total Score}} \times 100\% \quad [1]$$

The percentage obtained was then converted into quantitative criteria. Table 2 presents the percentage ranges used to determine these criteria.

Table 2: Percentage Range and Quantitative Criteria

Percentage (P)	Criteria
P > 80%	Very good
60% < P ≤ 80%	Good
40% < P ≤ 60%	Enough
20% ≤ P ≤ 40%	Not enough

Implementation

In the implementation phase, metacognitive scaffolding was applied to Graph Theory learning using the Problem-Based Learning (PBL) model, with the primary objective of improving students' mathematical creative thinking skills. To ensure comparability between the experimental and

control groups, both groups were taught the same Graph Theory topics within the same instructional period and were assessed using the same pretest and posttest instruments. The learning objectives, content coverage, duration of instruction and assessment procedures were maintained consistently across the two groups. The primary

instructional difference was that the experimental group received metacognitive scaffolding integrated into PBL activities through structured prompts for understanding problems, planning strategies, implementing plans, evaluating solutions and reflecting on the problem-solving process, whereas the control group received instruction without these structured metacognitive scaffolding prompts. Thus, the comparison was designed to examine the contribution of metacognitive scaffolding to students' mathematical creative thinking while maintaining comparable instructional conditions between the groups.

Evaluation

The evaluation phase is the product revision phase to improve the quality of the metacognitive scaffolding based on input and suggestions from expert examiners before testing it on students. After the implementation phase, an evaluation is conducted to determine whether the developed metacognitive scaffolding can improve students' mathematical creative thinking skills.

Data Collection

Data collection was conducted through an expert validation questionnaire, which included assessments of material, language, presentation, sample questions and practice questions. The researchers then used a test instrument to determine improvements in mathematical creative thinking skills.

Data Analysis

The data analysis technique uses quantitative descriptive. The data analyzed are the results of the posttest. Testing uses normality, homogeneity, practicality test and independent sample t-test.

Results

The metacognitive scaffolding developed in this study was designed based on identified patterns of students' mathematical creative thinking processes across different learning styles. The results of the analysis show that there are different mathematical creative thinking patterns for each subject with a particular learning style. Through thinking patterns, it was also found that there were obstacles when solving a problem in Graph Theory material. Thus, researchers developed metacognitive scaffolding as an effort to help students minimize obstacles to improving their mathematical creative thinking abilities.

Analysis

The analysis results show that students' mathematical creative thinking processes in solving graph theory problems are influenced by their learning styles in understanding, processing and representing them. Based on the Visual, Auditory and Kinesthetic learning style classification, students with a visual learning style tend to understand concepts more easily through pictorial and graphical representations, while auditory students are more helped by verbal explanations and discussions and kinesthetic students are more active in exploring solutions through hands-on activities. This finding aligns with research suggesting that differences in learning styles influence the characteristics of students' mathematical creative thinking abilities in solving mathematical problems (25).

Wallas's stages provide the framework for analyzing mathematical creative thinking processes in this study (26), which include preparation, incubation, illumination and verification. The characteristics of students' mathematical creative thinking are evident through indicators of fluency, flexibility, originality and elaboration, as evidenced by their ability to generate various alternative answers, use diverse strategies and develop solutions systematically and in detail (27). During the preparation stage, students are not yet able to write down the information they provide and ask for in detail and uniquely. During the incubation stage, students experienced difficulties understanding the concepts of paths, isomorphic graphs, bipartite graphs, simple graphs and chromatic numbers. During the illumination stage, some students experienced difficulties constructing isomorphic graphs, proving bipartite graphs, determining chromatic numbers and providing examples of simple graphs. Meanwhile, during the verification stage, students only checked the final answers without conducting in-depth evaluation and reflection on the problem-solving process. Based on these obstacles, this study developed a metacognitive framework encompassing problem understanding, planning strategies, implementing plans, evaluating and reflecting to help students improve their mathematical creative thinking skills in learning graph theory.

Metacognitive scaffolding plays a crucial role in supporting the development of students' mathematical creative thinking processes. Metacognition is a learning strategy that helps students plan, discuss and initiate their thinking processes during problem-solving. Through the provision of prompting questions, reflective guidance and step-by-step assistance, students become more aware of the strategies they use and can independently correct errors in their solutions. This finding is supported by previous research which states that metacognitive-based scaffolding is effective in improving creative thinking skills and mathematical problem-solving processes according to students' learning characteristics (28).

Design

The design stage was carried out to follow up on the analysis stage, namely designing metacognitive scaffolding. This scaffolding includes understanding the problem, planning a strategy, implementing the plan, evaluation and reflection.

Metacognitive scaffolding offers structured guidance that helps students regulate their learning by planning their activities, monitoring their progress and reflecting on their understanding and performance in completing specific tasks. Table 3 presents the different aspects of metacognitive scaffolding identified in previous studies.

Therefore, this study considers the metacognitive scaffolding aspects, including understanding the problem, planning a strategy, implementing the plan, evaluating and reflecting. These aspects align with the stages of the mathematical creative thinking process and are expected to assist students in creative thinking to solve graph problems through several structured stages. This scaffolding aims to stimulate and direct students toward creative thinking by utilizing metacognition, namely awareness and control of their own thought processes. The metacognitive scaffolding aspects in this study are shown in Figure 1

Table 3: Differences in Metacognitive Scaffolding Aspects

Aspect	Reference Number				
	(13)	(25)	(26)	(14)	(28)
First Aspect	Representing reasons for tasks and activities	Task analysis	Strategic planning	Planning	Understanding problems
Second Aspect	Presenting the relationship between feedback, goals and task objectives	Self-confidence / self-motivation	Concept	Monitoring	Building connections
Third Aspect	Supporting reflective thinking	Self-control	Mathematical models	Reflection	Formulating strategies
Fourth Aspect	Distinguishing between conclusions/facts and hypotheses	Self-observation	Calculation	Evaluation	Reflection and evaluation
Fifth Aspect	Monitoring understanding of the text	Self-assessment	Interpretation	-	-
Sixth Aspect	Focusing on the learning process	Self-reaction	-	-	-
Seventh Aspect	Fostering interaction and collaboration among participants	-	-	-	-

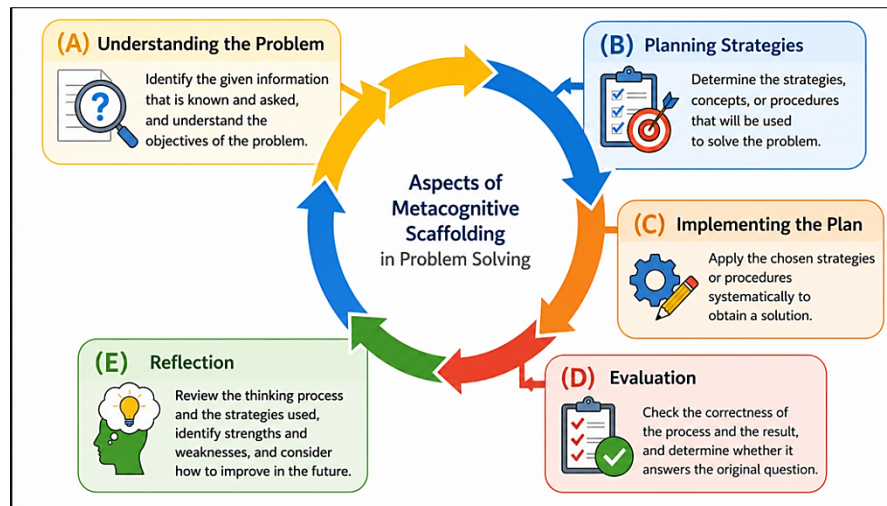


Figure 1: Aspects of Metacognitive Scaffolding in Problem Solving

Metacognitive scaffolding offers structured guidance that enables students to manage their learning by planning learning activities, integrating new knowledge and reflecting on their understanding of specific tasks (25, 26). The learning tools developed are in the form of textbooks and Student Worksheets. Textbooks and Student Worksheets are used to make it easier for students in the preparation, incubation, illumination and verification stages. This development was carried out in the Discrete Mathematics course, Graph Theory material.

Course Learning Outcomes include

- Students demonstrate an understanding of graph concepts and can differentiate various types of graphs;
- students can apply graph concepts in solving problems oriented to life skills. Students need scaffolding on path material, isomorphic graphs, bipartition graphs, simple graphs and chromatic numbers.

Table 4 presents the metacognitive scaffolding design used in this study.

Table 4: Metacognitive Scaffolding Design

Aspect	Metacognitive Scaffolding Instructions
Understanding the Problem	Students write information based on the problem given in the question. Students write down what is asked in the question. Students give reasons why the given problem is important.
Planning Strategy	Students write down the strategies that will be used to answer the problem.
Implementing the Plan	Students draw graphs based on information and complete with detailed descriptions. Students draw graphs that are isomorphic to known graphs and are accompanied by isomorphic proof. Students write alternative paths (paths) based on the problem, complete with information about the points and sides. Students identify a simple graph and provide reasons. Students prove a bipartition graph and give reasons. Students write the stages of the Welsh and Powel algorithms to determine the chromatic number in detail.
Evaluation	Students write alternative ways (if any) to present the solution. Students write a solution if the problem is changed to different conditions. Students write down whether there are any confusing steps. If there is, students need to explain how to overcome it.
Reflection	Students write down what they have learned based on the results of the solution. Students write down the challenging parts of the solution and give reasons. Students write down the strategies they will use if they encounter a similar problem.

Metacognitive scaffolding aspects are used to prepare student worksheets and examples of questions in textbooks used in graph theory learning in the Discrete Mathematics course. The learning model applied is the Problem Based Learning (PBL) model. The PBL model has a syntax that is relevant to the metacognitive scaffolding aspect, namely

a) Student orientation to the problem;

b) Organizing students to study;

c) Guide investigations independently and in groups;

d) Develop and present work results; and

e) Analyze and evaluate the process of solving a problem.

The graph learning design through the PBL model with Metacognitive Scaffolding is shown in Table 5 and Figure 2.

Table 5: Learning Design through the PBL Model with Metacognitive Scaffolding

Phase	Activity
Student orientation to the problem	The lecturer introduces the learning objectives and presents a Graph Theory problem. Students individually examine the problem, identify the known and required information and clarify the problem objectives using the understanding the problem prompts provided in the Student Worksheet. Aspects of understanding the problem: Identify known information, ask questions and understand the purpose of the problem.
Organizing students to study	The lecturer organizes students into groups and facilitates the identification of learning tasks. Students first consider possible strategies and then discuss them collaboratively within their groups to determine appropriate approaches. Students record the planned strategies in the Student Worksheet using the planning strategy prompts. Aspects of planning strategy: Write down strategies that can be used to answer the given problem.
Guide investigations independently and in groups	The lecturer facilitates the investigation and provides metacognitive prompts when needed. Students work collaboratively in groups to implement their planned strategies, explore alternative approaches, exchange ideas and construct solutions. They document their reasoning and solution procedures in the Student Worksheet as part of implementing the plan. Aspects of implementing the plan: Implement planned strategies systematically to obtain solutions.
Develop and present work results	The lecturer facilitates the presentation and provides feedback. Each group organizes and presents its solution to the class. Students compare alternative strategies, respond to questions and feedback and check the correctness of their reasoning and results using the evaluation prompts. Aspects of Evaluation: Checking the correctness of the process and settlement results.
Analyze and evaluate the process of solving a problem	The lecturer facilitates reflection on the problem-solving process. Students individually reflect on their thinking processes and strategies, identify strengths and difficulties and record what they learned and how their strategies could be improved using the reflection prompts in the Student Worksheet. The reflections may subsequently be discussed with the class. Aspects of Reflection: Review thought processes and strategies to gain better understanding.

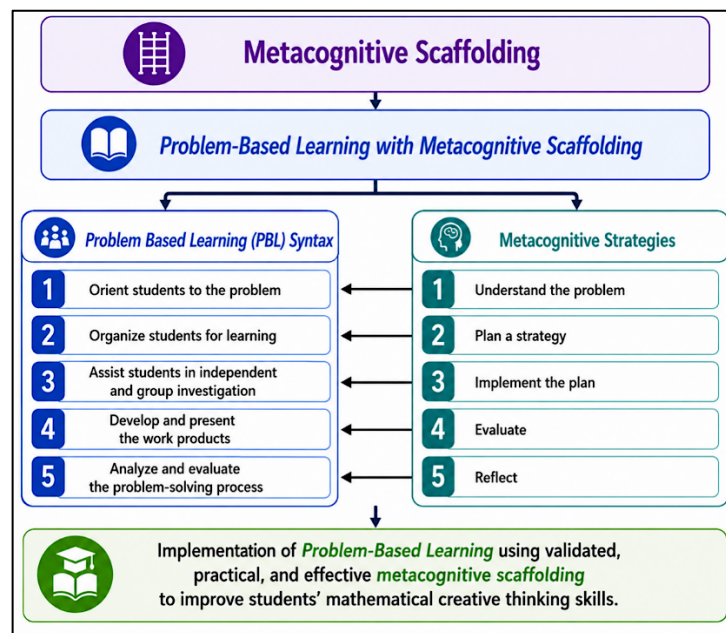


Figure 2: Integration of Metacognitive Scaffolding into Problem-based Learning (PBL)

Development

In the development stage, researchers developed Student Worksheets and Textbooks based on the designs produced in the previous stage. Development was carried out by applying metacognitive scaffolding aspects to the Student Worksheets and example questions in the textbook.

Student Worksheet with Metacognitive Scaffolding

Student Worksheets is part of a learning tool that contains course learning outcomes, learning objectives, work instructions, problems and completion stages. The problems given are in accordance with the results of qualitative data analysis, namely path material, isomorphic graphs, bipartition graphs, simple graphs and chromatic numbers. The beginning of the Student Worksheet contains aspects of understanding the problem by presenting a problem. Students are asked to

understand the problem then write down any information they know, ask questions and give reasons why it is important to understand the problem given. This is in accordance with the meaning of metacognitive scaffolding, namely a gradual assistance strategy provided by lecturers to help students develop the ability to think about their own thoughts (metacognition). The aim is for students to be able to realize what they know, plan how to solve problems, carry out plans, evaluate and reflect on results.

The development of Student Worksheets with metacognitive scaffolding has been validated by experts (expert judgment). This research involved four validators with material expert specifications (Graph Theory) and learning media experts (textbooks and Student Worksheets). Table 6 presents the expert validation results from the four validators.

Table 6: Validation of Student Worksheets

Aspect	Percentage (%)	Criteria	Improvement Suggestions	Revision Results
Material	98	Very good	Added answer key	Student Worksheets are equipped with answer keys.
Construction	95	Very good	Clarify scaffolding instructions according to Adapted Spelling and are unambiguous.	Scaffolding instructions have been clarified for each metacognitive aspect.
Language and Display	98	Very good	The problem language is made simple and easy to understand.	The problem language has been presented in language that is easy for students to understand.

The reflection stage is an important part of metacognitive scaffolding because it helps students become aware of the thought processes they engage in while solving mathematical problems. Through reflection, students not only review the result but also evaluate the strategies, difficulties and understanding gained during the problem-solving process. This reflection fosters metacognitive awareness, which includes the ability to plan, monitor and evaluate one's own thought processes. Research shows that metacognitive reflection plays a role in building awareness of tasks, strategies and the process of solving mathematical problems (29). In metacognitive scaffolding, instructors provide support in the form of reflective questions that guide students in identifying what they have learned, any difficulties they have encountered and strategies they will use for the next problem. This support helps students develop self-regulated learning and reflective thinking skills during the problem-solving process (30). The reflections provided by students on the worksheets demonstrate an awareness of the knowledge and skills they have acquired, such as converting

problems into graph form, determining bipartite graphs and determining chromatic numbers. Furthermore, students were able to identify the most challenging parts of the problem-solving process and explain the causes of these difficulties. During the planning stage, students demonstrated strategies they would reuse to solve similar problems in the future. This demonstrates that metacognitive scaffolding helps students develop reflective thinking and self-regulation skills in mathematical problem solving. Excerpt of student reflection in a metacognitive scaffolding worksheet can be seen in Figure 3.

Graph Theory Textbook with Metacognitive Scaffolding

The textbook being developed contains material on Graph Theory consisting of Definition and Terminology of Graphs, Types of Graphs, Point Degree, Graphic Presentation, Shortest Path and Graph Coloring. Each material is provided with example questions and solutions using metacognitive scaffolding aspects. The final stage is equipped with several evaluation questions that students can work on to improve their mathematical creative thinking abilities.

Section E. Reflection	
(A) What did you learn from the mathematical creative thinking process that you have carried out problems? I learned how to transform a problem into a graph, construct isomorphic graphs, identify several different paths, determine whether a graph is simple or not, identify bipartite graphs, and determine the chromatic number.	REFLECTION (Self-awareness) The student demonstrates awareness of what has been learned from the problem-solving process.
(B) Which part was the most challenging for you? Explain your reason. The most challenging part was transforming the problem into a graph and identifying bipartite graphs. In Graph M, the graph was not bipartite because the set of vertices could not be partitioned into two subsets.	MONITORING & EVALUATING (Monitoring and Evaluation) The student identifies the difficulties encountered during the problem-solving process and evaluates the causes of those difficulties.
(C) What strategy would you use in the future to solve similar problems? The strategy I would use is the same as the one I previously applied because it is easy to implement.	PLANNING (Planning) The student plans a strategy that will be used to solve similar problems in the future.

Figure 3: Excerpt of Student Reflection in a Metacognitive Scaffolding Worksheet

Graph theory is one of the fundamental branches of discrete mathematics that is widely used to model relationships and connections in real-world problems. One of the most classical problems in graph theory is the Königsberg Bridges problem introduced by Leonhard Euler, which later became the foundation for the development of graph theory. In mathematics learning, graph represen-

tation activities help students connect abstract concepts with visual structures, thereby strengthening conceptual understanding and problem-solving skills. Research has shown that students often experience difficulties in constructing graph representations and interpreting relationships between vertices and edges; therefore, instructional support in the form

of scaffolding is needed to assist students during the learning process (31).

One instructional approach that can support students' understanding is metacognitive scaffolding. This approach encourages students not only to solve problems but also to consciously monitor, evaluate and reflect on their own thinking processes. Metacognitive scaffolding has been reported to improve students' mathematical reasoning, critical thinking and conceptual understanding because students are guided through stages such as understanding the problem, planning strategies, implementing solutions, evaluating results and reflecting on the learning process (32). Therefore, integrating graph theory problems with metacognitive scaffolding is expected to help students develop both mathematical problem-solving abilities and self-regulated learning skills (31). Excerpt of example question in a graph theory textbook with metacognitive scaffolding can be seen in Figure 4. Textbook development with metacognitive scaffolding has been validated by experts (expert judgment). This research involved four validators with material expert specifications (Graph Theory) and learning media experts (textbooks and

Student Worksheets). Table 7 presents the expert validation results from the four validators.

Implementation

Learning implementation is carried out after the worksheet and textbook are declared suitable by the validator. The learning model applied is Problem Based Learning (PBL). This is because PBL has a syntax that is in accordance with the metacognitive scaffolding aspect. The research sample was taken from two classes consisting of students taking discrete mathematics courses.

Evaluation

The evaluation stage assessed the effectiveness of metacognitive scaffolding in graph learning using students' mathematical creative thinking test scores. The analysis compared the mean performance of students who learned with metacognitive scaffolding with that of students who received instruction without such support. Prior to hypothesis testing, the data were examined for normality and homogeneity to ensure that the statistical assumptions were satisfied. Differences between the two groups were then analyzed using an independent-samples t-test.

Consider the following diagram of the Königsberg bridges.

(A) Understanding the Problem

What is known?

- There are 7 bridges.
- There are 5 regions: A, B, C, D, and the outside region (E).

What is asked?

- Construct a graph that represents the Königsberg bridges.
- Determine whether the graph is simple and whether it is bipartite.
- Construct a graph that is isomorphic to the given graph.

(B) Planning Strategy

What strategy will you use?

- Represent each region as a vertex.
- Represent each bridge as an edge.
- Check for loops or multiple edges.
- Determine whether the graph is bipartite.
- Draw an isomorphic graph.

(C) Implementing the Plan

Graph representation of the Königsberg bridges:

Figure 4: Example Question in a Graph Theory Textbook with Metacognitive Scaffolding

Table 7: Validation of Textbooks

Aspect	Percentage (%)	Criteria	Improvement Suggestions	Revision Results
Material	98	Very good	Provides color for graph images that require color.	Textbooks already contain graphs with appropriate colors.

Language and Display	93	Very good	Add example questions for each material with metacognitive scaffolding solutions.	Each material in the textbook is equipped with example questions along with metacognitive scaffolding solutions.
Example Questions and Practice Questions	95	Very good	It is necessary to review the paper size and font in ISBN submissions.	Textbooks have adjusted the paper size and font for ISBN submissions.

Normality Test

The normality test was conducted to determine whether the sample data were derived from a normally distributed population. In this study, the Liliefors test was applied to the posttest data. Table 8 presents the results of the normality analysis.

The control class obtained $L_{count} = 0.0975$, while the experimental class obtained $L_{count} = 0.1063$. Both values were lower than the critical value $L_{table} = 0.1706$. Since $L_{count} \leq L_{table}$ for both groups, it can be concluded that the posttest data in the control and experimental classes were normally distributed. Therefore, the data satisfied

the normality assumption required for further parametric statistical analysis.

Homogeneity Test

The homogeneity test was performed to examine whether the variances of the populations from which the samples were drawn were equal. Table 9 presents the results of the homogeneity test.

The obtained values were $b_{count} = 1.2253$ and $b_k = 0.9246$. Based on the test results $b_{count} \geq b_k$, the significance level exceeded 0.05, indicating that the variances of the two groups were homogeneous. Thus, the data met the homogeneity assumption and were suitable for hypothesis testing using parametric statistics.

Table 8: Normality Test Results

Class	N	L_{count}	L_{table}	Information
Control	26	0.0975	0.1706	Normally Distributed
Experiment	26	0.1063	0.1706	Normally Distributed

Note: N = Number of Participants, L_{count} = Calculated Lilliefors Value, L_{table} = Critical Lilliefors Value.

Table 9: Homogeneity Test Results

Value	b_{count}	b_k	Information
Posttest	1.2253	0.9246	Homogeneous

Note: b_{count} = Calculated Bartlett Value, b_k = Critical Bartlett Value.

Practicality Test

The practicality test was conducted to assess the ease of implementing Problem-Based Learning (PBL) supported by metacognitive scaffolding through the Student Worksheets and Graph Theory Textbook. The practicality assessment was based on students' responses to a questionnaire covering four aspects:

- media,
- content,
- language and presentation and
- examples and practice exercises.

Each item was rated using a five-point Likert scale. The practicality percentage was calculated by comparing the total obtained score with the maximum possible score. The analysis yielded an

overall practicality score of 88.77%, which was classified as "Very Practical." This result indicates that the developed metacognitive scaffolding materials were considered easy to use and appropriate for supporting Graph Theory learning within the PBL framework.

Independent Sample t-test

The equality of means test was conducted to determine whether the mathematical creative thinking abilities of students in the experimental class were better than those in the control class. The equality of means test was analyzed using an independent sample t-test with the assistance of Microsoft Excel. Table 10 presents the calculation results.

Table 10: Independent Sample t-test Results

Class	<i>n</i>	$\bar{x}$	s^2	$s_{combined}$	t_{count}	t_{table}
Experiment	26	87	64.663	9.164	3.066	1.645
Control	26	79	103.282			

Note: *n* = Number of Participants, $\bar{x}$ = Mean, s^2 = Variance, $s_{combined}$ = Combined Standard Deviation, t_{count} = Calculated t-value, t_{table} = Critical t-value.

The results of the equality of means test showed $t_{count} = 3.066$. Based on the hypothesis testing criterion, H_0 is rejected if $t_{count} > t_{table}$. Since $t_{count} = 3.066 > t_{table} = 1.645$, it can be concluded that the mathematical creative thinking ability of students who received learning assisted by metacognitive scaffolding was significantly better than that of students who did not receive metacognitive scaffolding-assisted learning.

To complement the significance test, the magnitude of the difference was estimated using Cohen's *d*. Based on the group means and pooled standard deviation, the effect size was approximately $d = 0.87$, indicating a large effect. This result suggests that the difference between the experimental and control groups was not only statistically significant but also substantial in magnitude. Thus, students who received PBL integrated with metacognitive scaffolding demonstrated higher mathematical creative thinking performance than those who received instruction without structured metacognitive scaffolding. Taken together, the statistically significant difference and the large effect size provide evidence that the implementation of metacognitive scaffolding within PBL was effective in improving students' mathematical creative thinking in Graph Theory learning.

Discussion

The findings of this study indicate that the development of metacognitive scaffolding should be closely aligned with students' difficulties during mathematical creative thinking. The analysis revealed that students experienced difficulties in originality, flexibility and elaboration, especially during the incubation and illumination stages when solving graph theory problems. They also encountered difficulties in organizing relevant information, connecting concepts, constructing formal arguments and reflecting on their solutions. These findings suggest that students' difficulties are not solely related to conceptual knowledge but also to their ability to regulate their thinking during problem solving. This finding is consistent with previous research showing that scaffolding

can support learning achievement and promote students' independence in solving mathematical problems (28). However, whereas previous studies have generally examined scaffolding in relation to overall problem-solving performance, the present study identifies scaffolding needs based on specific difficulties that emerge during students' mathematical creative thinking processes. This distinction may be explained by the characteristics of graph theory, which require students to coordinate abstract concepts, visual representations, logical reasoning and alternative solution strategies.

Based on these identified difficulties, the metacognitive scaffolding developed in this study comprises five aspects: understanding problems, planning strategies, implementing plans, evaluation and reflection. These aspects were systematically integrated into the PBL phases rather than provided as separate learning activities.

These five metacognitive processes may support mathematical creative thinking through different but interconnected mechanisms. Understanding the problem helps students identify relevant information, relationships and possible interpretations, thereby providing a basis for generating multiple ideas and supporting fluency. Planning strategies encourages students to consider different representations and alternative approaches before selecting a solution pathway, thereby promoting fluency and flexibility and creating opportunities for originality. During plan implementation, students develop and articulate their selected strategies systematically, which supports elaboration by requiring them to provide detailed reasoning and solution procedures. Evaluation enables students to monitor the effectiveness of their strategies, identify approaches that are not working and compare alternative solutions, thereby encouraging flexibility and refinement of ideas. Finally, reflection encourages students to reconsider, improve, or extend their strategies and solutions, potentially supporting originality and elaboration.

Thus, these metacognitive processes do not correspond exclusively to individual dimensions of creativity; rather, they interact iteratively to support fluency, flexibility, originality and elaboration throughout mathematical problem solving.

This finding is consistent with previous research indicating that metacognitive scaffolding in collaborative settings facilitates questioning, investigation, peer interaction and productive knowledge construction (29). Similarly, previous studies have reported that PBL integrated with evaluation, reflection and peer assessment can enhance students' creative thinking (30). However, the present study extends these findings by embedding metacognitive guidance throughout the PBL process and aligning it with students' identified creative-thinking difficulties. This structured integration may be particularly relevant in graph theory because students need to move continuously between interpreting problems, constructing graphical representations, selecting strategies, generating alternative solutions and verifying their reasoning. Thus, the contribution of the intervention may arise not simply from the use of PBL or scaffolding independently, but from the alignment among PBL activities, metacognitive regulation and the cognitive demands of graph theory.

The expert validation results indicate that the developed textbooks and Student Worksheets provide appropriate instructional support for implementing this framework. Rather than interpreting the high validation scores merely as evidence of technical quality, these results suggest alignment among graph theory content, metacognitive prompts, learning activities and the intended mathematical creative thinking processes. Expert recommendations concerning clearer scaffolding instructions, additional worked examples, simplified problem statements, answer keys and improved graph visualization further strengthened this alignment. This finding supports previous research demonstrating that student-centered metacognitive scaffolding can enhance conceptual understanding and facilitate effective problem-solving processes (31). Nevertheless, the present study differs in that metacognitive prompts were explicitly incorporated into both textbooks and Student Worksheets based on previously identified difficulties in students'

creative thinking processes. In the context of this study, such explicit guidance may be particularly important because students initially demonstrated limited ability to articulate strategies, develop alternative ideas and reflect independently on their solutions.

The practicality assessment yielded a score of 88.77%, categorizing the developed Graph Theory Textbook and Student Worksheets as "Very Practical." This result indicates that the learning materials were perceived positively in terms of media, content, language and presentation and examples and practice exercises. The high practicality may be attributed to the structured metacognitive prompts embedded in the materials, which guided students through understanding problems, planning strategies, implementing plans, evaluating solutions and reflecting on their thinking. This finding is consistent with previous research indicating that structured metacognitive scaffolding can promote student engagement, self-regulation and independence in problem solving. In the present study, the integration of these prompts into PBL activities provided practical guidance for both individual and collaborative Graph Theory problem solving, supporting the feasibility of the developed materials for classroom implementation.

The effectiveness findings further demonstrate that students who learned through PBL supported by metacognitive scaffolding achieved better mathematical creative thinking performance than students who learned without such scaffolding. Importantly, the effect size analysis yielded Cohen's $d = 0.87$, indicating a large effect. This suggests that the difference between the experimental and control groups was not only statistically significant but also substantial in magnitude. The large effect provides additional evidence that the structured integration of metacognitive scaffolding into PBL may have educationally meaningful benefits for students' mathematical creative thinking in Graph Theory. This result is consistent with previous research showing that metacognitive learning can lead to better mathematical performance than conventional instruction(32). However, earlier research has generally emphasized mathematical achievement, analytical ability, or general problem-solving performance, whereas the

present study focuses specifically on mathematical creative thinking in graph theory. This difference is important because creative problem solving requires not only obtaining correct answers but also generating multiple ideas, using flexible strategies, producing original solutions and elaborating reasoning. A possible explanation for the improved performance is that the scaffolding prompts made students' thinking processes more explicit. Prompts for understanding problems and planning strategies may have encouraged students to consider different approaches, while evaluation and reflection may have prompted them to reconsider, refine and justify their solutions. These processes are particularly relevant to graph theory, where problems frequently allow multiple representations and alternative solution pathways.

Although the present findings are generally consistent with previous studies (28–32), some differences can be identified in the scope and instructional context. Previous studies have largely examined scaffolding, metacognitive learning, or PBL as separate approaches or have focused on broader outcomes such as achievement, problem solving and analytical skills. In contrast, the present study integrates these components within a single instructional framework specifically designed from students' mathematical creative thinking characteristics. Therefore, the findings should not be interpreted as indicating that metacognitive scaffolding or PBL alone produces the observed improvement. Rather, the results suggest that their systematic alignment with students' creative-thinking needs may be an important factor. The graph theory context may also contribute to the observed outcomes because its visual, relational and non-routine characteristics naturally provide opportunities for generating and evaluating alternative mathematical ideas.

Overall, this study extends previous research by demonstrating how metacognitive scaffolding can be systematically designed, operationalized through a Graph Theory Textbook and Student Worksheets and integrated into PBL to support mathematical creative thinking. The contribution of this study therefore extends beyond the development of learning materials; more importantly, it provides a structured instructional design comprising five interconnected metacognitive processes understanding problems, planning

strategies, implementing plans, evaluation and reflection embedded within the PBL phases. Nevertheless, the findings should be interpreted within the specific context of Graph Theory learning and the characteristics of the participating students. Differences in Mathematical topics, student characteristics and instructional environments may produce different outcomes. Future research should therefore examine the proposed design with larger and more diverse samples, extend its application to other mathematical topics and investigate its long-term effects on mathematical creative thinking.

Conclusion

This study developed a metacognitive scaffolding design integrated into Problem-Based Learning (PBL) for Graph Theory learning. The scaffolding comprises five interconnected aspects: understanding problems, planning strategies, implementing plans, evaluation and reflection, implemented through a Graph Theory Textbook and Student Worksheets. The findings showed that the developed metacognitive scaffolding was valid, practical and effective. The practicality assessment indicating that the developed materials were highly practical for classroom implementation. Furthermore, the experimental group achieved a higher mean mathematical creative thinking score than the control group. The independent-samples t-test confirmed a significant difference between the groups and the effect size indicated a large effect. These findings provide evidence supporting the effectiveness of metacognitive scaffolding integrated into PBL for enhancing mathematical creative thinking in Graph Theory.

The main contribution of this study is a structured metacognitive scaffolding design that systematically integrates five metacognitive aspects into the PBL phases. This design provides practical guidance for lecturers to facilitate students' mathematical creative thinking through structured metacognitive support. However, the study was limited by its relatively small sample and specific higher-education context. Future research should examine the proposed design with larger and more diverse samples, extend its application to other mathematical topics and investigate its long-term effectiveness.

Abbreviations

ADDIE: Analysis, Design, Development, Implementation and Evaluation, PBL: Problem Based Learning.

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Author Contributions

Noviana Dini Rahmawati: Data Collection, Initial Stage Article Preparation, Budi Waluya: Literary Studies, Final Stage Article Preparation, Mulyono: Preparation of Scaffolding Material, Discussion of Research Results, Iqbal Kharisudin: Research Data Analysis, Research Data Presentation.

Conflict of Interest

The authors declare no competing interests related to this research.

Data Availability

Study data are available from the corresponding author upon reasonable request but are not publicly accessible due to participant privacy and confidentiality considerations.

Declaration of Artificial Intelligence (AI) Assistance

ChatGPT (OpenAI) was used to improve the language, clarity and readability of the manuscript and to assist in creating visual presentations based on the authors' concepts and specifications. It was not used for data analysis, interpretation of findings, or formulation of scientific conclusions. All AI-assisted content was reviewed and verified by the authors, who take full responsibility for the accuracy and integrity of the manuscript.

Ethics Approval

This paper was specifically submitted for submission to this journal and is not currently under consideration elsewhere. This research complies with established ethical guidelines and has received university approval to conduct this work honestly and ethically. The parties involved have voluntarily agreed to participate.

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